Using SMT Solvers (to reason about programs)

Martin Kellogg

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B. equality of uninterpreted functions
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Agenda: SMT solvers

- **Motivation: reasoning about formulas**
- SAT solving: DPLL
- SMT solving: Nelson-Oppen and DPLL(T)
- SMT in practice: brief intro to Z3 and SMT-LIB

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    - goal: create test cases that definitely increase coverage
- At the time, we deferred the question of how we would solve path predicates **automatically**
  - recall that a ***path predicate*** is a formula over program variables that is true when a particular path is executed

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For example, consider this program:

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int simpleMath(int a, int b) {
  assert(b > 0);
  if(a + b == a * b) {
    return 1;
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Key question: are there a, b **such that this is true**?

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- In our lecture on symbolic execution, I briefly mentioned that ***SMT solvers*** are the modern tool that we'd use to do so
  - let's do it now: https://www.philipzucker.com/z3-rise4fun/

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- Other applications **include**:
  - reasoning about program correctness (automating pen-and-paper proofs!)
  - reasoning about program equivalence (cf. equivalent mutant problem)
  - program synthesis
  - program repair
  - etc.

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- different solvers might support different theories
  - much of today's reading was about various theories that Z3 supports, such as ***Equality of Uninterpreted Functions*** (EUF) and the theory of Arrays

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- note that in the symbolic execution case, we're interested in the satisfying assignment (it's the test case)
  - in many other interesting cases, we want to check a formula's **validity**: that is, whether it is true for all values of its inputs

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This means that we can use an SMT solver to check either **validity** or **satisfiability**!

- useful for e.g. proving program equivalence

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    - and was also the main subject of today's reading
  - hopefully you will also get a sense for **when and when not** to apply an SMT-based tool

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- Motivation: reasoning about formulas
- **SAT solving: DPLL**
- SMT solving: Nelson-Oppen and DPLL(T)
- SMT in practice: brief intro to Z3 and SMT-LIB

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Review: basics of SAT

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Example boolean formulas:

- $a \lor b \land \neg c$
- $(P \land Q) \lor (Q \land \neg R)$
- etc.

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    - no: there is no choice of X that makes both X and $\neg X$ true at the same time

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- Naïve solution: try all possible assignments
  - Takes $O(2^n)$ time for a formula with *n* variables (**slow**!)

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- So how do they manage to be so fast when the underlying problem is so hard?
  - We'll discuss two core algorithms:
    - the **DPLL** algorithm for efficiently solving SAT
    - the **Nelson-Oppen** algorithm for efficiently solving SMT

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Example CNF formulas:

- $(a \lor b) \land (\neg c)$
- $(a \lor \neg b) \land (\neg a \lor c) \land (b \lor c)$

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- if the input formula is not in CNF, we can **transform it into CNF automatically** via DeMorgan's laws, the double negative law, and the distributives laws over boolean operators
  - I'm not going to cover this, because you should have had a discrete math class before. If you can't confidently do this now, you should practice before the exam.

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    - for a one-literal clause to be true, that literal must be true!

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$$(a \lor b) \land (\neg c) \land (\neg a \cancel{\lor c}) \land (b \cancel{\lor c})$$

- $\neg c$ appears alone, so c must be false

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Continuing the example:

(a ∨ b) ∧ (¬c) ∧ (¬a ~~∨ c~~) ∧ (b ~~∨ c~~)

- b only appears positively, so we can set it to true

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  - generally you can pick whatever variable you'd like if I ask you to do DPLL (e.g., on an exam) when you are stuck
- it is important to **remember** what you guessed
  - if you reach an **unsatisfiable** result, you need to ***backtrack*** to the point where you made the guess (and try the other option)

DPLL: algorithm

```
function DPLL(Φ)
    // unit propagation:
    while there is a unit clause {l} in Φ do
        Φ ← unit-propagate(l, Φ);
    // pure literal elimination:
    while there is a literal l that occurs pure in Φ do
        Φ ← pure-literal-assign(l, Φ);
    // stopping conditions:
    if Φ is empty then
        return true;
    if Φ contains an empty clause then
        return false;
    // DPLL procedure:
    l ← choose-literal(Φ);
     return DPLL(Φ ∧ {l}) or DPLL(Φ ∧ {¬l})
```

pseudo-code from https://en.wikipedia.org/wiki/DPLL_algorithm

DPLL: algorithm

Heuristic: try **unit propagation** first because it creates more units and pure literals.

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DPLL: algorithm

Pure literal elimination is tried second because it only eliminates entire clauses (it can’t create new units or pure literals).

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Fallback: try both assignments to a random literal. (Note the short-circuiting “or” operator.)

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DPLL: putting it all together

Try to do DPLL in pairs on the following formula:

$(a \lor b) \land (a \lor c) \land (\neg a \lor c) \land (a \lor \neg c) \land (\neg a \lor \neg c) \land (\neg d)$

From SAT to SMT

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- For the moment, we will assume the existence of solvers for these **theories** (such as linear arithmetic)
  - but note that separate satisfying assignments for two theories might not be compatible!
- Core idea of SMT: solve theories **separately**, but use DPLL to combine them (called ***DPLL(T)***)

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- **Key idea**: replace expressions from each theory with variables
  - variables introduced by Nelson-Oppen can be shared between theories
  - solve the whole formula with a modified variant of DPLL, then ask the theory solvers if the satisfying assignment makes sense

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Let's use the following formula as an example:

$f(f(x) - f(y)) = a \quad \land \quad f(0) = a + 2 \qquad \land \quad x = y$

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- ***arithmetic***:

SMT: Nelson-Oppen

At this point in class, I tried to solve this example on the board. I got it wrong; it is not satisfiable. See next week's slides.

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Note how theories communicate using (only) **equalities**

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- ***equality of uninterpreted functions*** (***EUF***): *f*(e1) = *a*, e2 = *f*(*x*), e3 = *f*(*y*), *f*(e4) = e5, *x* = *y*
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  - Continue until done

SMT: DPLL(T) example

Consider this formula as an example:

$x >= 0 \land y = x + 1 \land (y > 2 \lor y < 1)$

Conveniently all clauses are in **linear arithmetic**, so we can skip purification

SMT: DPLL(T) example

Consider this formula as an example:

$$x >= 0 \wedge y = x + 1 \wedge (y > 2 \vee y < 1)$$

$$p1 \wedge p2 \wedge (p3 \vee p4)$$

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$$p1 \land p2 \land (p3 \lor p4)$$

We now solve this with DPLL. We get a satisfying assignment (e.g., p1, p2, p4 all true). Then, we check this with our theory:

- can p1, p2, and p4 all be true at the same time?

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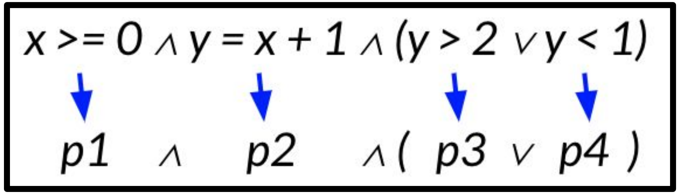
  - add **new clause** $(\neg p1 \lor \neg p2 \lor \neg p4)$, try again

SMT: DPLL(T) example



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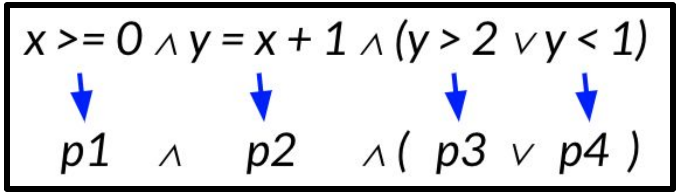

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Run DPLL again; one satisfying assignment is p1, p2, p3, ¬p4

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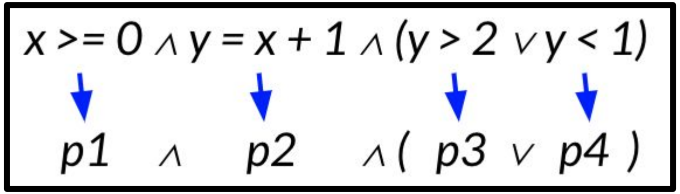

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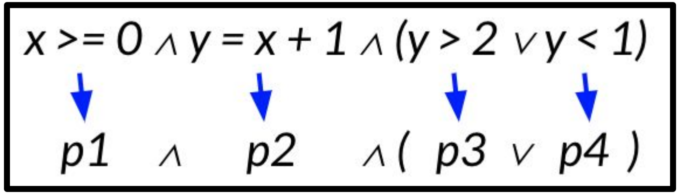

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- check these again against our theory. Can these all be true at the same time?
- **yes!**
    - So, we’re done.

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  - setting the variable for x > 10 to true will make x < 7 false!
- DPLL(T) must support **adding clauses to the formula**
  - to represent the knowledge gained from theories

Agenda: SMT solvers

- Motivation: reasoning about formulas
- SAT solving: DPLL
- SMT solving: Nelson-Oppen and DPLL(T)
- **SMT in practice: brief intro to Z3 and SMT-LIB**

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  - **printing to the screen**

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(echo “Running Z3”)
(declare-const a Int)
(assert (> a 0))
(check-sat)
(get-model)
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What question does this code answer?

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What question does this code answer?
"Does an integer greater than 0 exist?"

SMT in practice: a more complex example

Consider this code:

```
int getNumber(int a, int b, int c) {
  if (c == 0) return 0;
  if (c == 4) return 0;
  if (a + b < c) return 1;
  if (a + b > c) return 2;
  if (a * b == c) return 3;
  return 4;
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Suppose we want to know if the **pink** statement is ever executed. What constraints should we pass to the SMT solver to check?

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Su

the

executed. What constraints should we pass to the SMT solver to check?

All of the following must be true:

- !(c == 0)
- !(c == 4)
- !(a + b < c)
- !(a + b > c)
- a * b == c

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  if (a + b > c) return 2;
  if (a * b == c) return 3;
  return 4;
}
```

Su
the
executed. What constraints should we pass to the SMT solver to check?

All of the following must be true:

- !(c == 0)
- !(c == 4)
- !(a + b < c)
- !(a + b > c)
- a * b == c

Let's turn this into code for [the solver](#)!

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- Z3 also supports reasoning about **bit vectors** of fixed size
  - let’s model Java ints (32 bits) and ask the same question…
    - it terminates quickly!
    - finite search space

Another example: program equivalence

Consider these two programs:

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int add1(int a, int b) {
  return a + b;
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- does this match our intuition?
- what have we **actually proven**?

Proving universal claims

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  - “proving no counter-examples” via SMT solver means that we’re looking for **unsat** as an answer
    - need to phrase the question to the solver as “does there exist an input such that these programs differ”
      - if it says “no” (=unsat), then the programs are the same!

Let’s try with Z3 again, this time changing our question to ask if there are counter-examples.

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  - Support many theories that can model program semantics.
  - Usually support a standard language (SMT-lib).
- The challenge is to model a problem as a constraint system.
- Many higher-level DSLs and language bindings exist.
  - but in HW10 you'll mostly use SMT-LIB directly

Course announcements

- Next week's topic will be **DevOps**
    - I have already posted the required readings
- I will soon send out a survey about when you'd like to do a **final exam review**
    - reminder: the final exam is on May 9th at 6pm (here)