More Static Semantics

Martin Kellogg

Agenda

- Finish discussion of subtyping/let from last class
- More type rules
- Method dispatch rules
  - static
  - dynamic
- SELF_TYPE

Course Announcements

- Don’t ignore PA2!
    - Listen to the TAs when they tell you to start ASAP

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Semi-review: Assignment Rule

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- What is "Γ"? "⊢"? "≤"?

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- The following program does ***not*** typecheck:

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- Why not? **Typing environment** hasn't been updated!

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- Now consider another hypothetical **wrong** let rule:

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- How is this one different from the correct rule?
- The following ***bad program (!)*** is well-typed:

  let x : B <- new A in x.b()

- Why is this program bad?

Examples of Wrong Let Rule (3)

- Now consider another hypothetical **wrong** let rule:

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- How is this one different from the correct rule?
- This "good" program is not well-typed:

  let x : A <- new B in { … x <- new A ; x.a(); }
- Why isn't this program well-typed?

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- But no matter how well we choose the type rules, ***some good programs will be rejected anyway***
  - Rice's Theorem strikes again: typechecking is **undecidable**

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$$\frac{\Gamma_C(\mathbf{id}) = \mathbf{T_0} \qquad \Gamma_C \vdash \mathbf{e_1} : \mathbf{T_1} \qquad \mathbf{T_1} \leq \mathbf{T_0}}{\Gamma_C \vdash \mathbf{id} \texttt{ <- } \mathbf{e_1} : \mathbf{T_0}\,;} \quad \textbf{[Attr-Init]}$$

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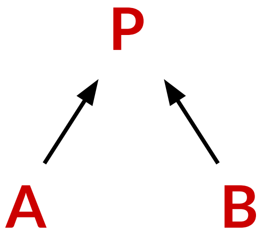

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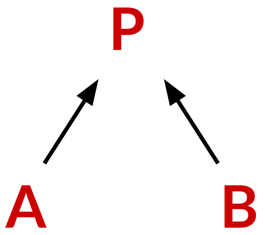

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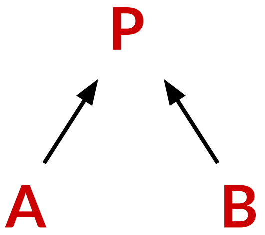

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- In Cool, the least upper bound of two types is their **closest common ancestor** in the inheritance tree (= type hierarchy)

If-Then-Else Rule

[If-Then-Else]

If-Then-Else Rule

$$\frac{\Gamma \vdash e_0 : \mathbf{Bool} \quad \Gamma \vdash e_1 : T_1 \qquad \Gamma \vdash e_2 : T_2}{\Gamma \vdash \mathbf{if}\ e_0\ \mathbf{then}\ e_1\ \mathbf{else}\ e_2\ \mathbf{fi} : \mathit{lub}(T_1, T_2)} \ \text{[If-Then-Else]}$$

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$$\frac{\Gamma \vdash e_0 : T_0 \qquad \Gamma[T_1/x_1] \vdash e_1 : T_1' \qquad \ldots \qquad \Gamma[T_n/x_n] \vdash e_n : T_n'}{\Gamma \vdash \textbf{case } e_0 \textbf{ of } x_1 : T_1 \texttt{ => } e_1 \,;\, \ldots \,;\, x_n : T_n \texttt{ => } e_n \,;\, \textbf{esac}\textbf{:}\ \mathit{lub}(T_1', \ldots, T_n')}\ \textbf{[Case]}$$

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- We need information about the **formal parameter types** and **return type** of **f**, but Γ doesn't contain that information!

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    - $\mathbf{M}$ maps a (class, method) tuple to a method signature
    - e.g., $\mathbf{M}(C, f) = (T_1, \ldots, T_n, T_{ret})$ means that there is a method in class **C** with the signature $\mathbf{f}(x_1 : T_1, \ldots, x_n : T_n) : T_{ret}$

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- We read this as "with the assumptions that the free object identifiers in e have the types given by Γ *and the method identifiers in e have the signatures given by* $\mathbf{M}$, the expression e has type T."

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- Only the **Dispatch** rule actually uses $\mathbf{M}$.

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$$\frac{\begin{array}{ll} \Gamma, \mathbf{M} \vdash e_0 : T_0 & \\ \Gamma, \mathbf{M} \vdash e_1 : T_1 & \mathbf{M}(T_0, f) = (T_1', \ldots, T_n', T_{n+1}') \\ \quad \ldots & \\ \Gamma, \mathbf{M} \vdash e_n : T_n & \forall \text{ i in } (1\ldots n), T_i \leq T_i' \end{array}}{\Gamma, \mathbf{M} \vdash e_0.f(e_1, \ldots, e_n) : T_{n+1}'} \text{ [Dispatch]}$$

Method Dispatch Revisited

first check receiver object e_0

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Method Dispatch Revisited

then check actual arguments are well-typed

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Method Dispatch Revisited

four steps!

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3. next, look up method signature...

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- Cool also supports ***static dispatch***
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- The static dispatch rule is similar to the dynamic dispatch rule, but the inferred type of the receiver must conform to the type that the programmer specifies

(new additions/changes from the regular Dispatch rule in purple)

Static Dispatch Rule

$$\frac{\begin{array}{ll} \Gamma, \mathbf{M} \vdash e_0 : T_0 & T_0 \leq T \\ \Gamma, \mathbf{M} \vdash e_1 : T_1 & \mathbf{M}(T, f) = (T_1', \ldots, T_n', T_{n+1}') \\ \quad \ldots & \\ \Gamma, \mathbf{M} \vdash e_n : T_n & \forall\ i \text{ in } (1 \ldots n),\ T_i \leq T_i' \end{array}}{\Gamma, \mathbf{M} \vdash e_0@T.f(e_1, \ldots, e_n) : T_{n+1}'} \text{ [Static Dispatch]}$$

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- An active line of research: inventing more flexible type systems while preserving soundness
  - SELF_TYPE is an "advanced" feature, to give you a taste of this

Review: Dynamic and Static Types

- Define the ***dynamic type*** of an object as ???

- Define the ***static type*** of an expression as ???

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- Define the *dynamic type* of an object as the class C that is used in the “new C” expression that creates the object in some execution
    - run-time notion, present even in languages without static types
- Define the ***static type*** of an expression as the *least upper bound* of the dynamic types that the expression can take on, in some execution
    - cf. static vs dynamic semantics

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    - subclasses can **only add** attributes or methods

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    - subclasses can **only add** attributes or methods
    - methods can be redefined, but **only with the same types**

An Example

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class Count {
  i : Int <- 0;
  inc() : Count {
    {
      i <- i + 1;
      self;
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  - …or does it?
- There is a **problem lurking** here!

Co(u)ntinuing the Example

- Consider a subclass **Stock** of **Count**:

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class Stock inherits Count {
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our current rules will cause a typechecking error here, because inc() returns a Count (not a Stock)

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- But this is not well-typed, because `(new Stock).inc()` has *static* type **Count**
- The typechecker has "lost" information

Trivia Break: History

This city on the Danube river famously was the home of a number of influential figures of the 20th century for a short time in 1913, including Leon Trotsky, Joseph Stalin, Adolf Hitler, Sigmund Freud, and Josip Broz Tito. It was the seat of the Holy Roman Emperors of the Habsburg dynasty from the 16th-century until the empire's dissolution in 1806 (with only brief interruptions). Afterward, it was the seat of Austria-Hungary until the dissolution of that empire following the first World War. It is now the capital of Austria.

Trivia Break: Computer Science

This American computer scientist and mathematician was the recipient of the 1974 Turing Award. He has been called the "father of the analysis of algorithms". He is the author of the multi-volume work *The Art of Computer Programming*. In addition to his work in theoretical computer science, he is the creator of the TeX computer typesetting system, the related METAFONT font definition language and rendering system, and the Computer Modern family of typefaces.

SELF_TYPE to the Rescue

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  - In the case of `(new Stock).inc()`, the type is **Stock**
- We introduce the keyword **SELF_TYPE** to use for the return value of such functions
  - We will need to modify the type rules to handle SELF_TYPE

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- The program from before is now well typed

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  - SELF_TYPE is a static type
- It helps the typechecker to keep better track of types
- It enables the typechecker to accept more correct programs
- In short, having SELF_TYPE increases the **expressive power** of the type system

SELF_TYPE and Dynamic Types (Example)

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- In general, if **SELF_TYPE** appears textually in the class C as the declared type of E then it denotes the dynamic type of the “self” expression:

  dynamic_type(E) = dynamic_type(self) ≤ C
- Note: The meaning of **SELF_TYPE** depends on where it appears
  - We write **SELF_TYPE**$_C$ to refer to an occurrence of **SELF_TYPE** in the body of some specific class C

Typechecking SELF_TYPE

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- What's **wrong** with this?
  - It's sound, but it's like not having **SELF_TYPE** at all (oops)

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Typechecking SELF_TYPE (properly)

- Recall the operations that we've defined over types:
  - subtyping: $T_1 \leq T_2$
  - least upper bound: $\mathit{lub}(T_1, T_2)$
- To handle **SELF_TYPE** properly, we need to **extend** these operations to handle it
  - doing so is surprisingly involved…

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    - Thus this is the most flexible rule we can allow
2. $\text{SELF\_TYPE}_C \leq \text{SELF\_TYPE}_C$
    - recall that SELF_TYPE_C is the type of the “self” expression
    - In Cool we *never* need to compare **SELF_TYPE**s coming from different classes (why? left as an exercise…)

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    - Why?

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- Using these rules, we can extend ***lub*** too…

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  3. *lub*(T, **SELF_TYPE$_C$**) = *lub*(**C**, T)
     - bonus question: why is this the same as case 2?
  4. *lub*(T, T') defined the same as before

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    - No: the `T` in static dispatch needs to refer to a *specific, dynamic type*

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$$\Gamma, \mathbf{M}, \mathsf{C} \vdash \mathsf{e} : \mathsf{T}$$

- Read as "An expression **e** occurring in the body of C has static type **T** given a variable type environment Γ and method signatures **M**"

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- E.g.,:

$$\frac{\Gamma, M, C \vdash e_1 : T_1 \quad \Gamma(id) = T_0 \quad T_1 \leq T_0}{\Gamma, M, C \vdash id \texttt{ <- } e_1 : T_1} \text{ [Assign]}$$

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- There are a number of other places in the rules where SELF_TYPE appears - read the CRM carefully

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```
class A { comp(x : SELF_TYPE) : Bool {...}; };
class B inherits A {
  b() : int { ... };
  comp(y : SELF_TYPE) : Bool { ... y.b() ...}; };
...
let x : A  new B in ... x.comp(new A); ...
...
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  - SELF_TYPE as the return type in an invoked method might have ***nothing to do*** with the current class

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- But it is **illustrative** of a class of ideas that trade-off expressiveness for complexity
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