Typechecking and Static Semantics

Martin Kellogg

Today’s Agenda

- Typing Rules
- Typing Environments
- “Let” Rules
- Subtyping
- Wrong Rules

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We’ll start by reviewing some of what we saw at the end of the last lecture…

English to Inference Rules

If $\mathbf{e_1}$ has type **Int** and $\mathbf{e_2}$ has type **Int**,
then $\mathbf{e_1 + e_2}$ has type **Int**

Building blocks:

- $\wedge$ is "and"
- **->** is "if-then"
- **x : T** is "x has type T"

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($\mathbf{e_1}$ has type **Int** $\wedge$ $\mathbf{e_2}$ has type **Int**) ->
$\mathbf{e_1 + e_2}$ has type **Int**

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$(\mathbf{e_1} : \mathbf{Int} \land \mathbf{e_2} : \mathbf{Int}) \rightarrow \mathbf{e_1 + e_2} : \mathbf{Int}$

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Traditional notation (same meaning!):

$$\frac{\vdash e_1 : \mathbf{Int} \qquad \vdash e_2 : \mathbf{Int}}{\vdash e_1 + e_2 : \mathbf{Int}}$$

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(e_1 has type **Int** ∧ e_2 has type **Int**) ->
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(e_1 : **Int** ∧ e_2 : **Int**) -> $e_1 + e_2$: **Int**

Building blocks:

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$$\frac{\vdash e_1 : \mathrm{Int} \qquad \vdash e_2 : \mathrm{Int}}{\boxed{\vdash} e_1 + e_2 : \mathrm{Int}}$$

Pronounced "we can prove that..."

Inference Rule Examples

$$\frac{\vdash e_1 : \mathrm{Int} \qquad \vdash e_2 : \mathrm{Int}}{\vdash e_1 + e_2 : \mathrm{Int}}\ [\mathrm{Add}]$$

$$\frac{\textit{i is any integer constant}}{\vdash i : \mathrm{Int}}\ [\mathrm{Int}]$$

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- These rules give templates describing how to type integers and + expressions
- By filling in the templates, we can produce complete typings for expressions

Baby's First Type Derivation

$$\dfrac{}{\vdash \mathbf{1 + 2} : \mathbf{Int}} \; \textbf{[Add]}$$

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Baby’s First Type Derivation

“ground facts” (I will write these in *italics*)

$$\dfrac{\dfrac{\underline{\textit{1 is an integer constant}}}{\vdash 1 : \mathrm{Int}}\,[\mathrm{Int}] \qquad \dfrac{\underline{\textit{2 is an integer constant}}}{\vdash 2 : \mathrm{Int}}\,[\mathrm{Int}]}{\vdash 1 + 2 : \mathrm{Int}}\,[\mathrm{Add}]$$

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- We only want sound rules, but some sound rules are worse than others
  - e.g., consider this rule:

$$\frac{\textit{i is an integer constant}}{\vdash \mathbf{i} : \mathbf{Object}}\ \textbf{[Int-Obj]}$$

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$$\frac{\vdash \mathbf{e_1} : \mathbf{Int} \qquad \vdash \mathbf{e_2} : \mathbf{Int}}{\vdash \mathbf{e_1} + \mathbf{e_2} : \mathbf{Int}}\ \textbf{[Add]}$$

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Next, we're going to look at a collection of **examples** of type rules

subexpression proofs

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Rules for Constants

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Notation note: I'm using **bold black** for **keywords**, **bold blue** for **expressions**, and **bold red** for **types**

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- new T produces an object of type T
  - ignore SELF_TYPE for now...
- That gives us this rule:

$$\frac{}{\vdash \textbf{new } T : T}\ \textbf{[New]}$$

Rules for Bools and Loops

$$\frac{\vdash \mathbf{e} : \mathbf{Bool}}{\vdash \mathbf{not}\ \mathbf{e} : \mathbf{Bool}}\ [\mathbf{Not}]$$

Rules for Bools and Loops

$$\frac{\vdash e : \text{Bool}}{\vdash \text{not } e : \text{Bool}} \text{ [Not]}$$

$$\frac{\vdash e_1 : \text{Bool} \qquad \vdash e_2 : T}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \text{ [Loop]}$$

Typing Example

- Typing for `while not false loop 1 + 2 * 3 pool`

`while loop pool`

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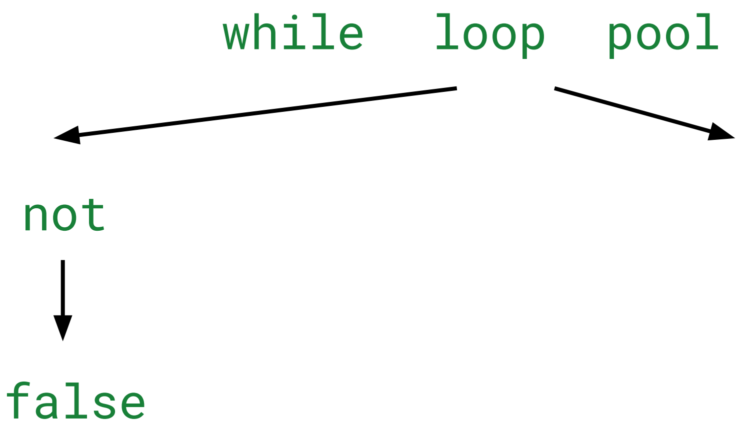

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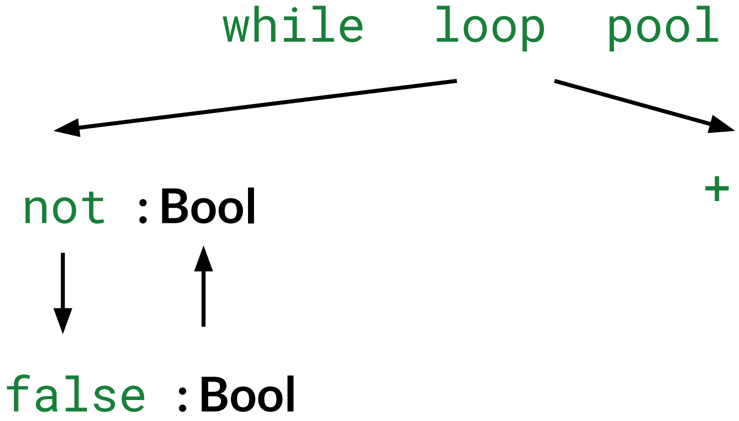

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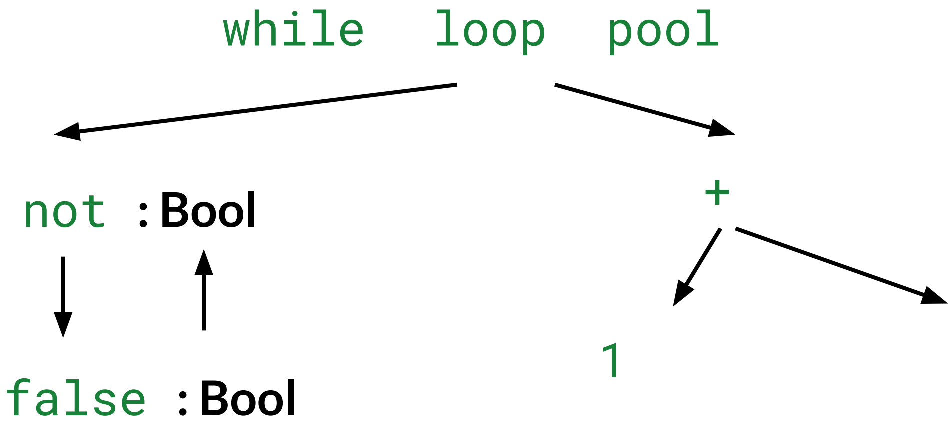

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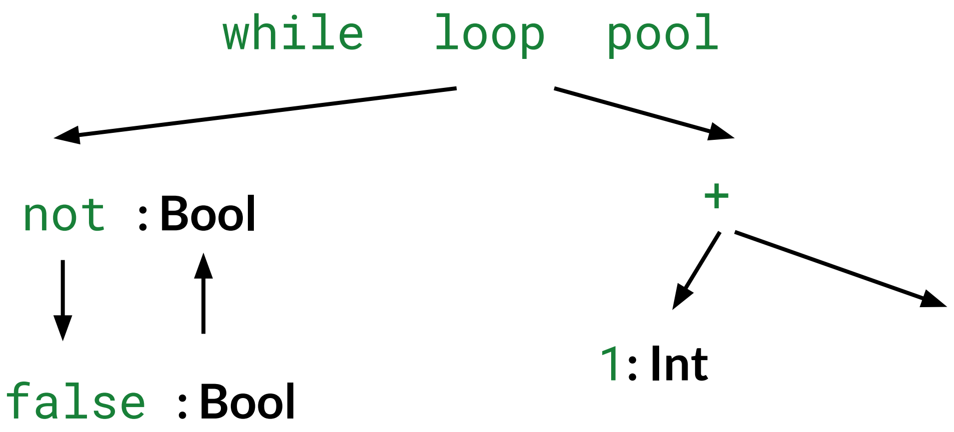

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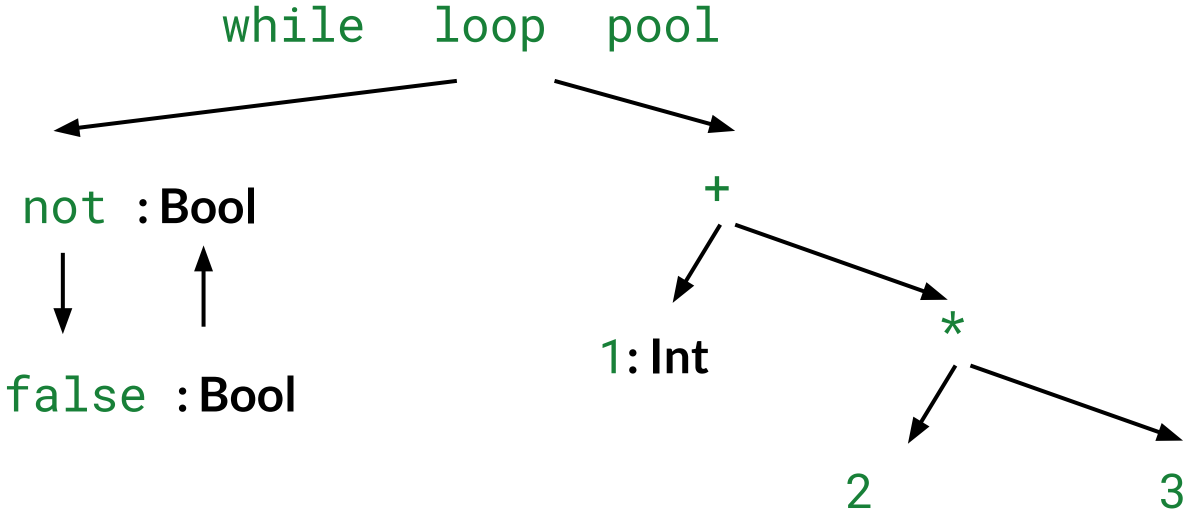

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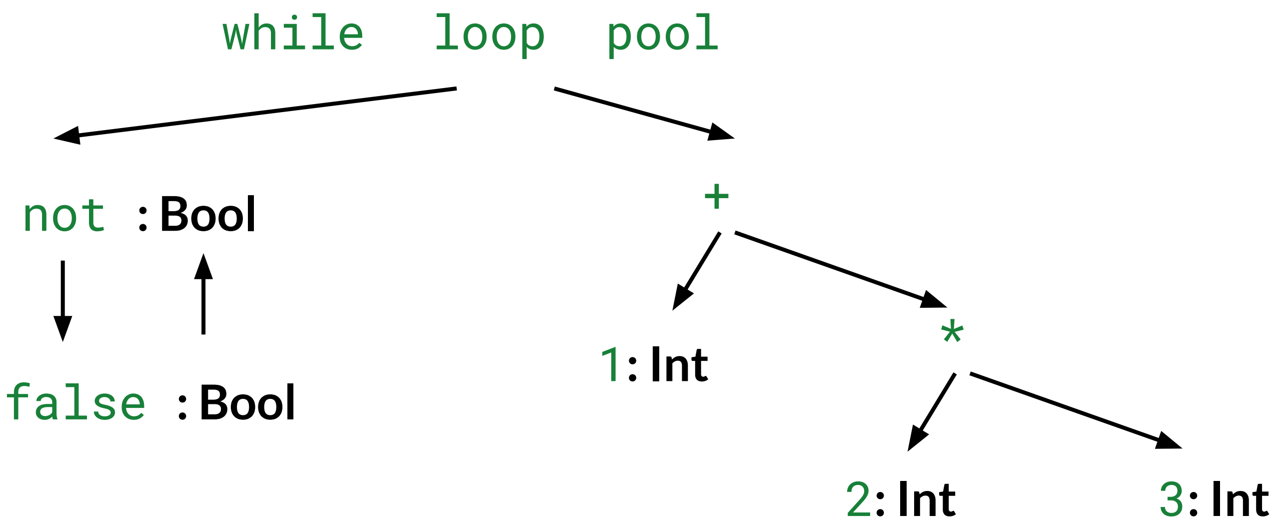

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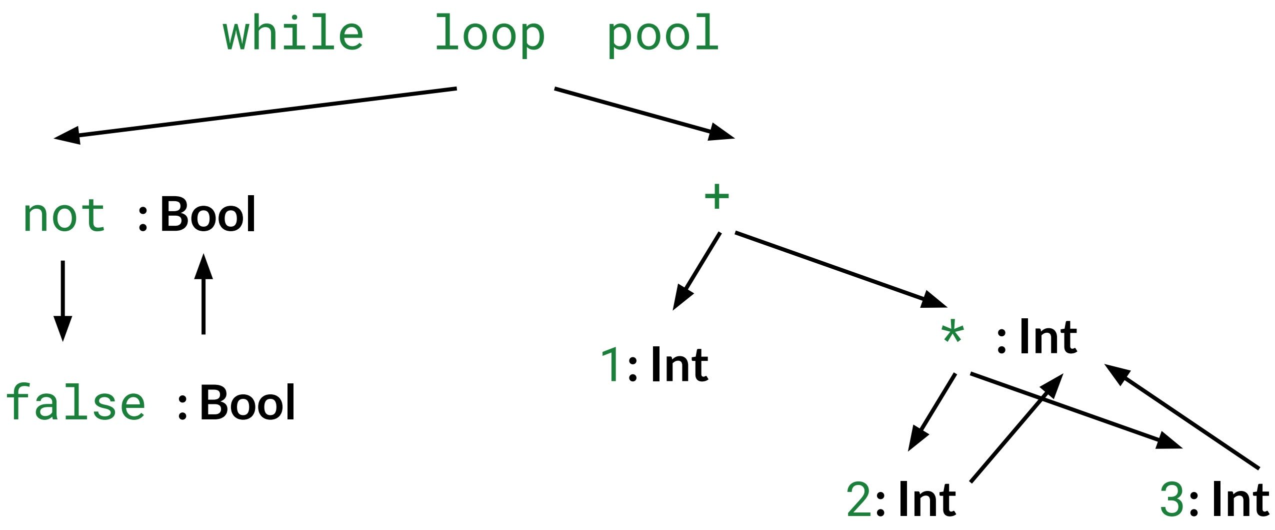

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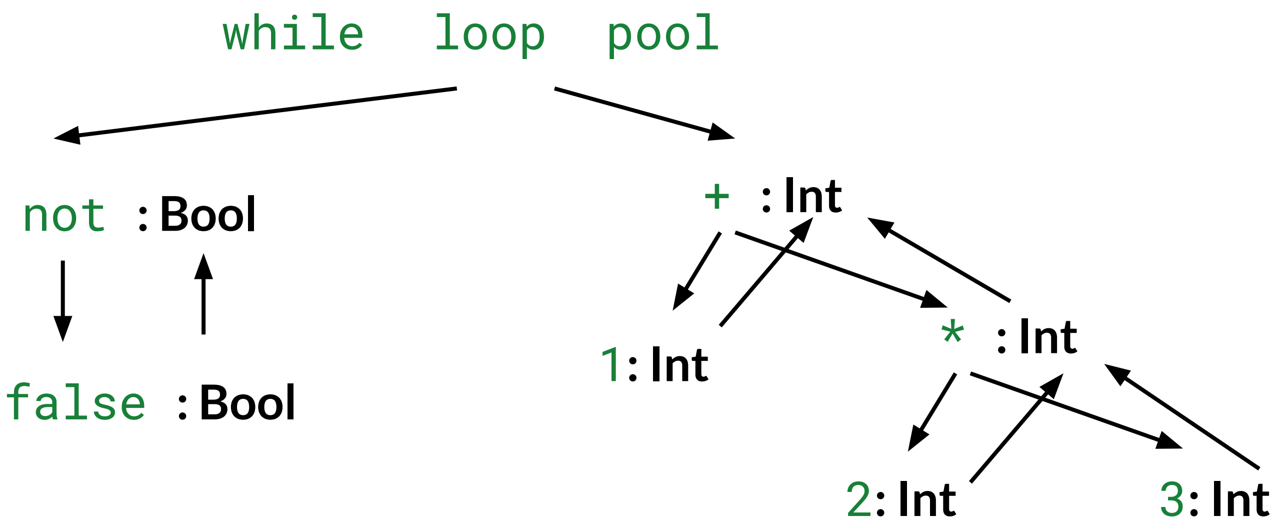

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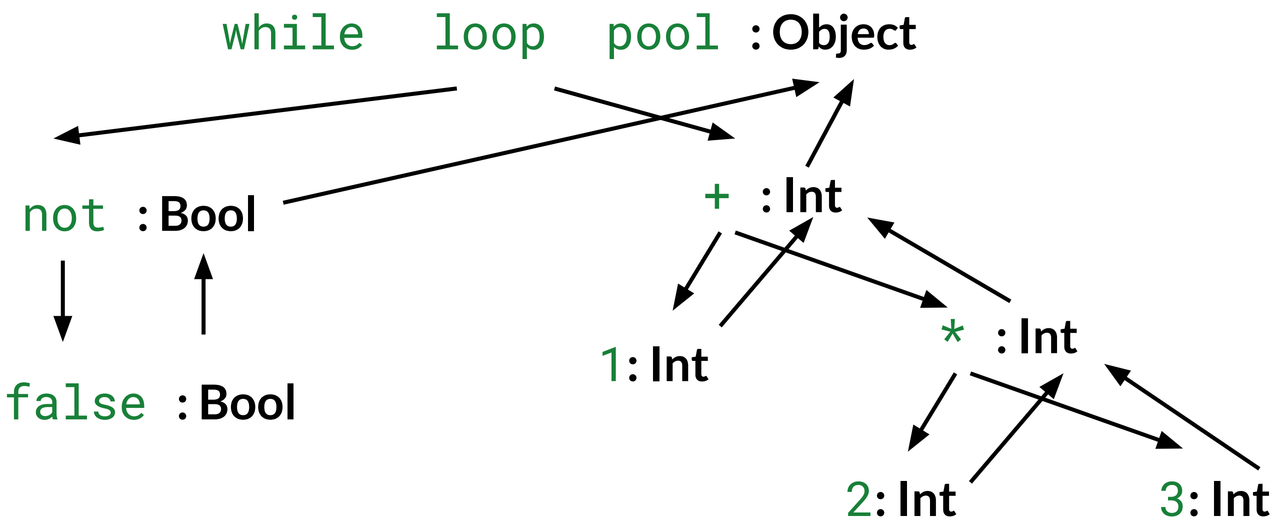

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- **Leaves** in the tree are the rules with ground facts

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- This local structural rule does **not** carry enough information to give x a type (oh no)
    - All of the rules we've looked at so far have been about **constants**…

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expression: “x”
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> "Γ" is a **capital gamma** (the third letter of the Greek alphabet). We use Γ for type environments by convention.

Modified Rules

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- More formally:

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- $\Gamma[\mathbf{T}/\mathbf{x}]$ means "Γ modified to map $\mathbf{x}$ to $\mathbf{T}$, and behaving as Γ on all other arguments"
- More formally: $\Gamma[\mathbf{T}/\mathbf{x}](\mathbf{x}) = \mathbf{T} \land \Gamma[\mathbf{T}/\mathbf{x}](\mathbf{y}) = \Gamma(\mathbf{y}) \ \forall \ \mathbf{y} \neq \mathbf{x}$

New Rules

$$\frac{x \text{ is an identifier} \qquad \Gamma(\mathbf{x}) = \mathbf{T}}{\Gamma \vdash \mathbf{x} : \mathbf{T}} \; \textbf{[Var]}$$

You can write $\Gamma[\mathbf{x}/\mathbf{T}]$ on tests, etc., without penalty.

- We're almost ready for today's main event: the **let rule**
- First, we need one more piece of notation: $\Gamma[\mathbf{T}/\mathbf{x}]$
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Explanation:

- if, after replacing x's old type with T_0, we can prove that e_1 has type T_1...
- ...then the **let** expression is well-typed by T_1

Let Example

- Consider this Cool expression:

  $\textbf{let } x : T_0 \textbf{ in } (\textbf{let } y : T_1 \textbf{ in } E_{x,y}) + (\textbf{let } x : T_2 \textbf{ in } F_{x,y})$

  (where $E_{x,y}$ and $F_{x,y}$ are some Cool expressions that contain occurrences of “x” and “y”)

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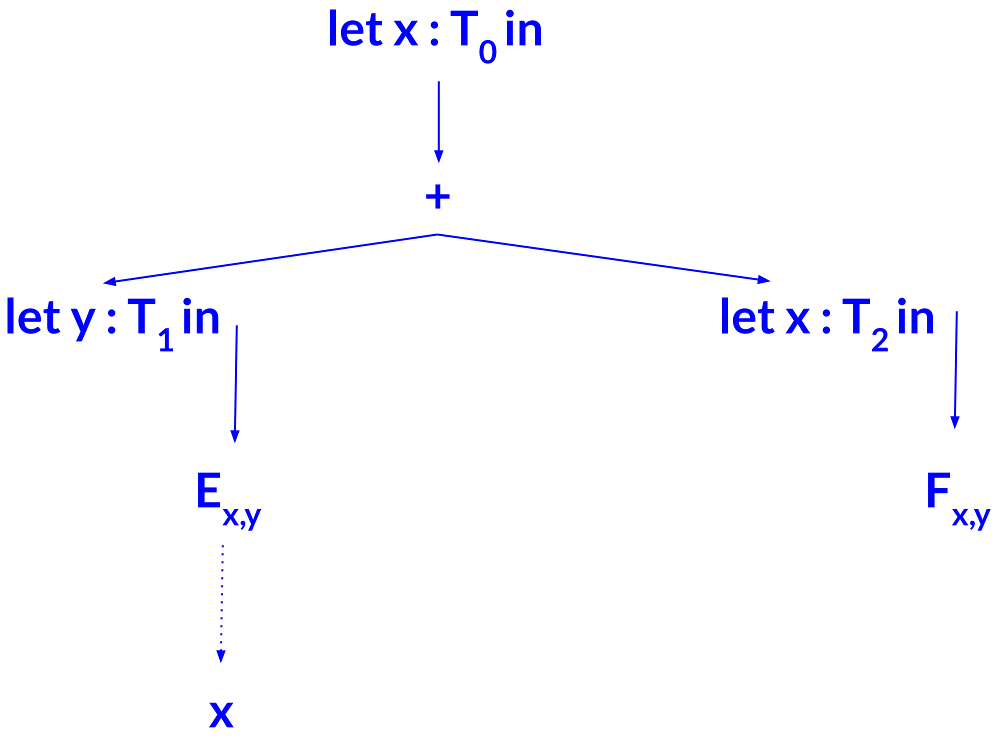
  - "Γ[T/x]"-like replacements **exactly match** the scoping we'd expect!

Example of Typing Let

AST



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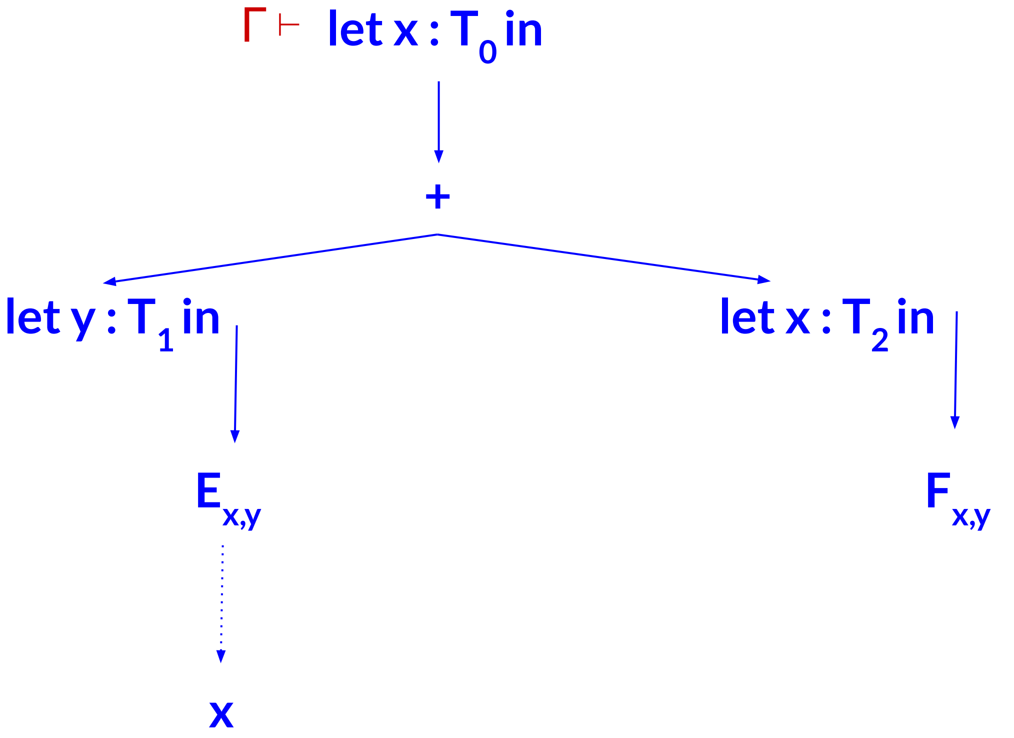
AST
Type env.



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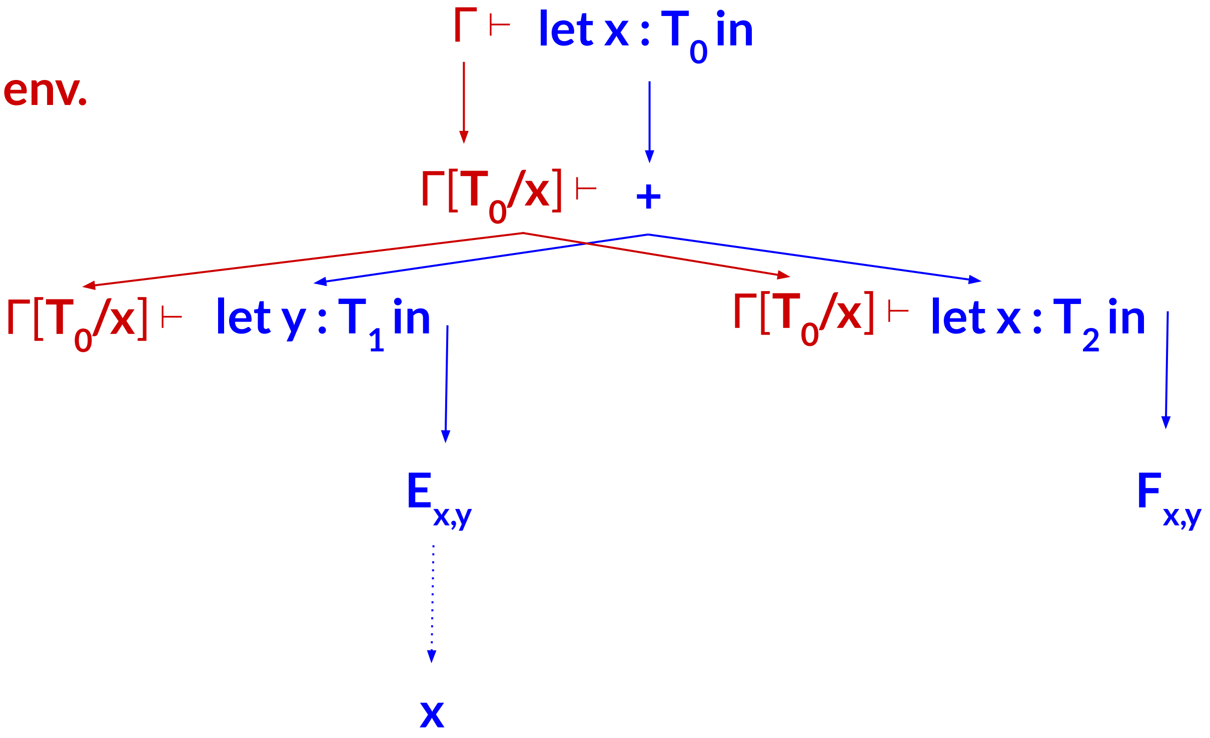

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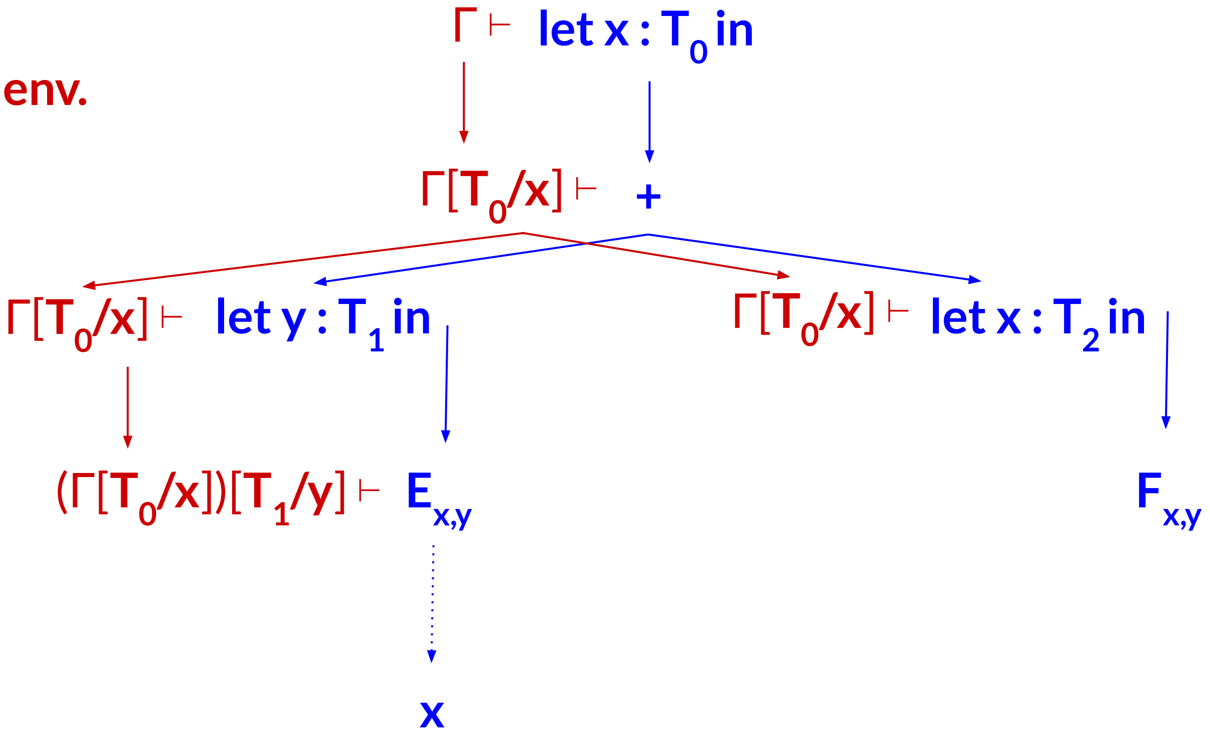

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$\Gamma \vdash$ **let** $x : T_0$ **in**

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$\Gamma[T_0/x] \vdash$ **let** $y : T_1$ **in**

$\Gamma[T_0/x] \vdash$ **let** $x : T_2$ **in**

$(\Gamma[T_0/x])[T_1/y] \vdash E_{x,y}$

$F_{x,y}$

$(\Gamma[T_0/x])[T_1/y] \vdash x$

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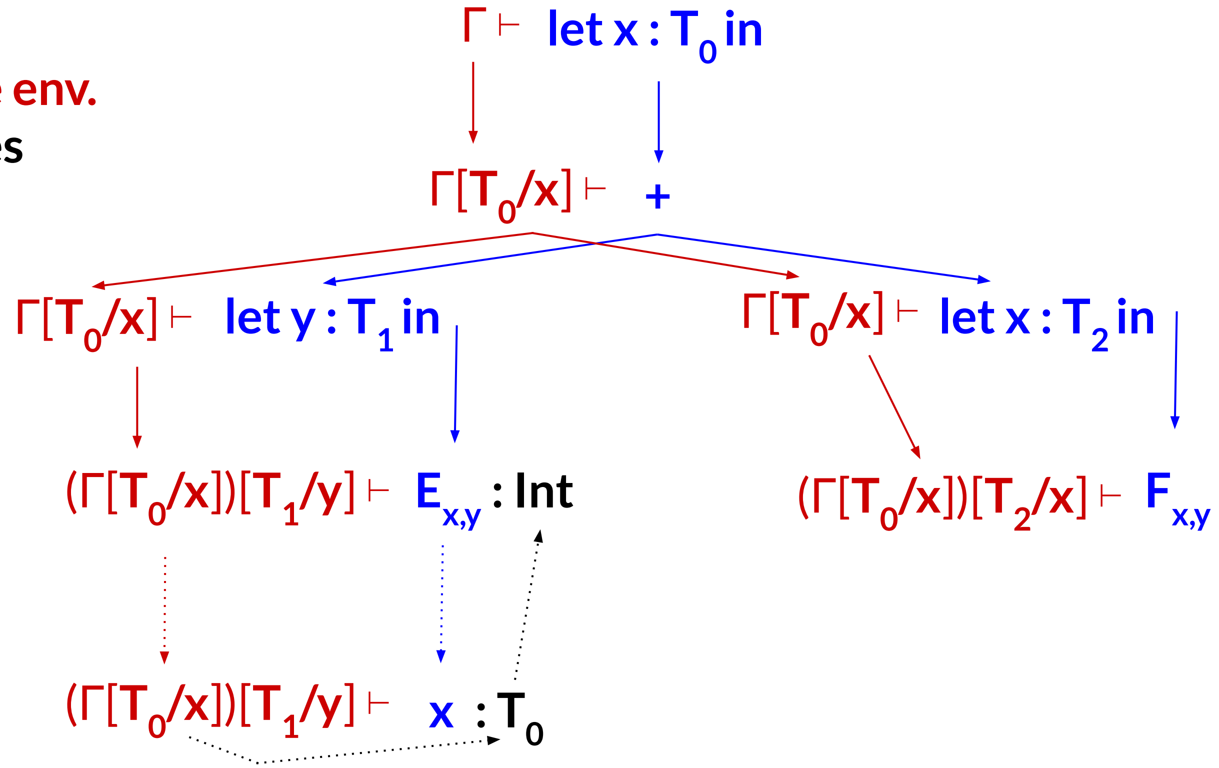

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$$\Gamma[T_0/x] \vdash \text{let } y : T_1 \text{ in} \quad : \text{Int} \qquad \Gamma[T_0/x] \vdash \text{let } x : T_2 \text{ in}$$

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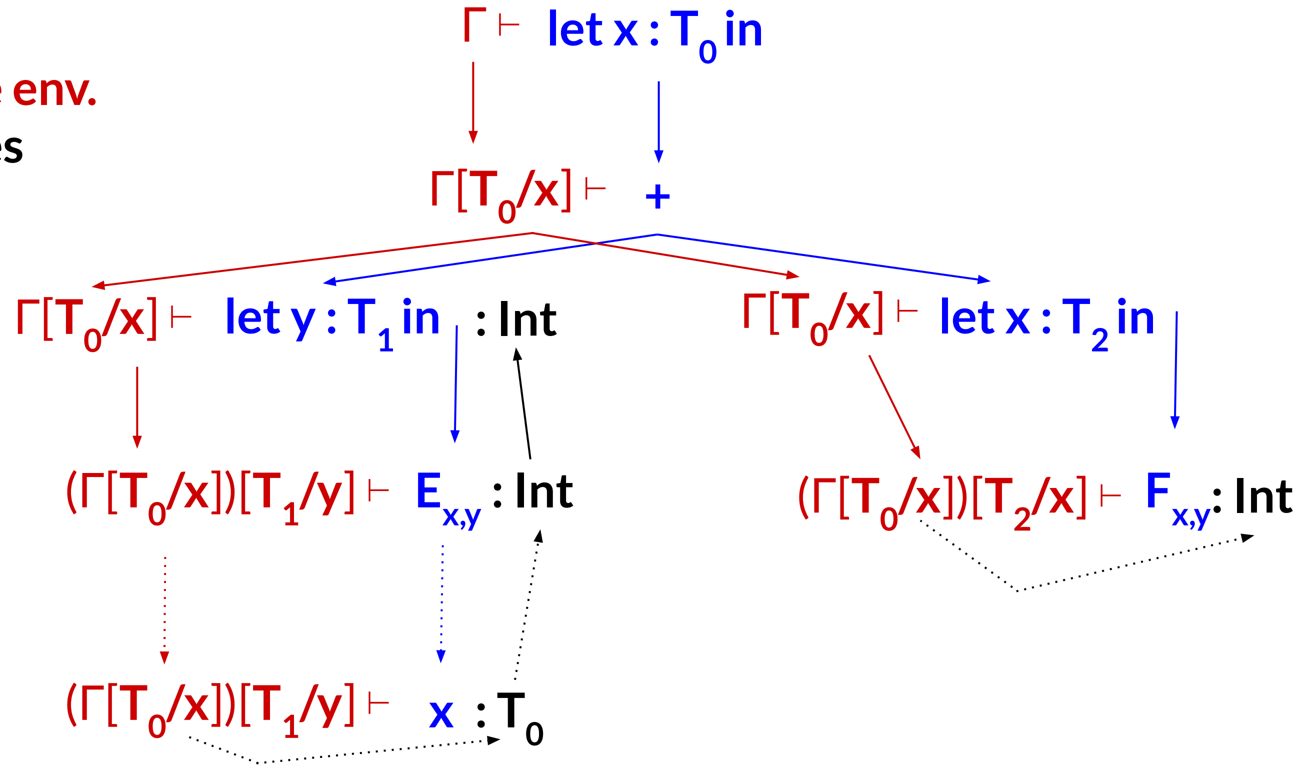

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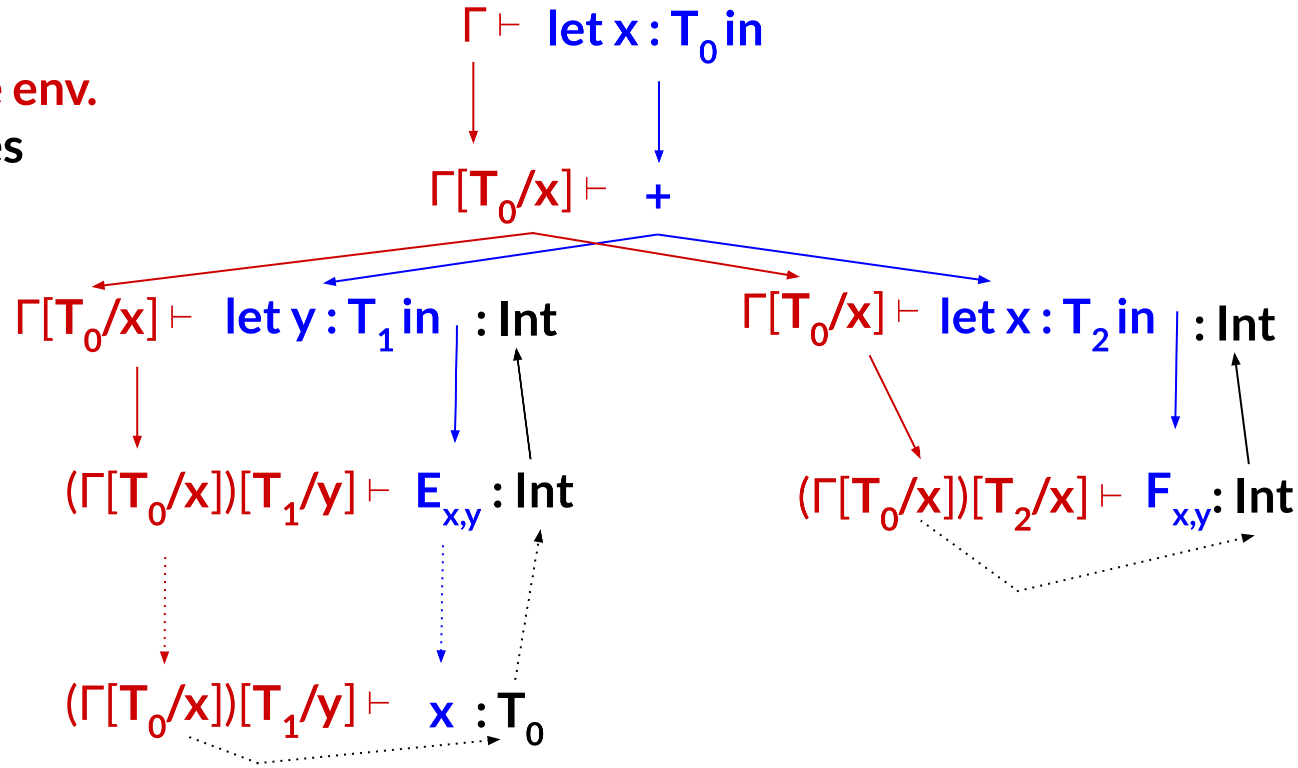

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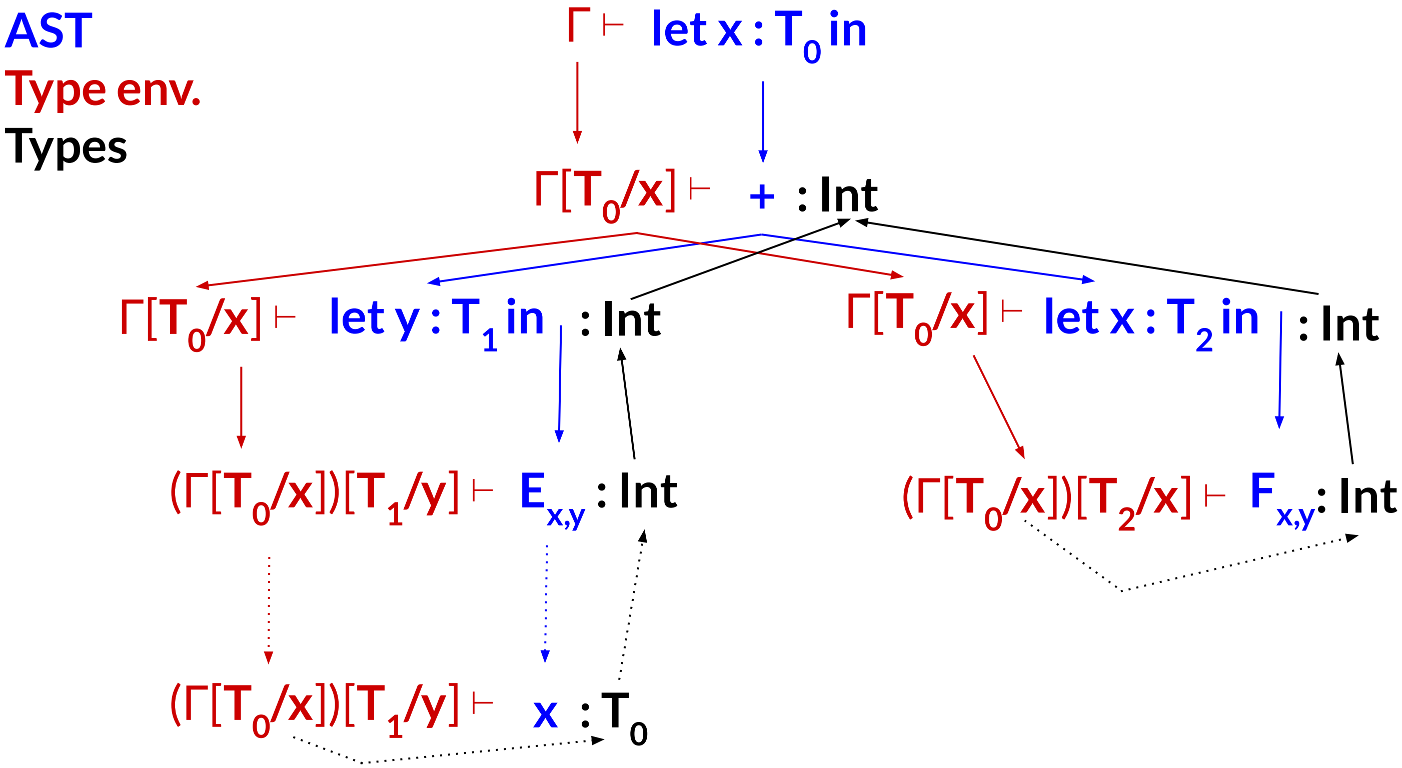

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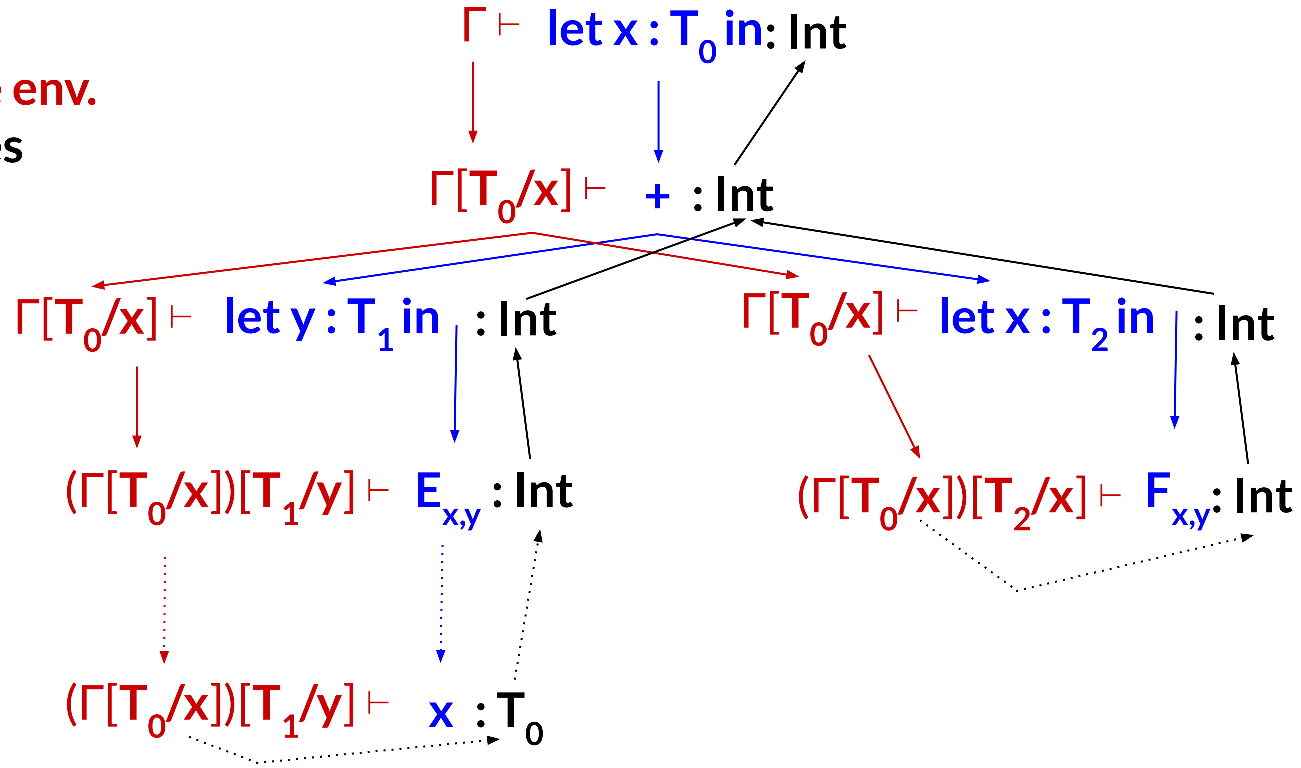

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- What would the typing derivation be? Get out a piece of paper. I'll get you started...

$$\frac{???}{\Gamma \vdash 1 + \textbf{let x : Int in x + 2} : ???}$$

Type Environment Notes

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- The type environment is **passed down** the AST from the root towards the leaves
- Types are computed **bottom-up** on the AST from the leaves **toward the root**

Trivia Break: History

This book documented the environmental harm caused by the indiscriminate use of DDT, a pesticide used by soldiers during World War II. It was met with fierce opposition by chemical companies, but it swayed public opinion and led to a reversal in US pesticide policy, a nationwide ban on DDT for agricultural uses, and an environmental movement that led to the creation of the US Environmental Protection Agency. The book's author is Rachel Carson.

Let with Initialization

- Now consider let **with initialization**:

$$\frac{\Gamma \vdash e_0 : T_0 \qquad\qquad \Gamma[T_0/x] \vdash e_1 : T_1}{\Gamma \vdash \textbf{let } x : T_0 \texttt{ <- } e_0 \textbf{ in } e_1 : T_1} \textbf{[Let-Init]}$$

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- This rule is **weak**. Why?

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class C inherits P { ... }
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- The proposed let rule **does not allow** this code (because "new C" (e_0) does not exactly have the type "P" (T_0))
  - We say that a rule is ***too weak*** or ***incomplete*** when it prevents us from typechecking "good" programs (but Rice's Theorem...)

Subtyping

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Subtyping

Reminder from math class: a ***relation*** is some specific subset of the Cartesian product of some finite list of sets

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  - In Cool, this means that **X** is a ***subclass*** of **Y**
- Definition of **≤** on classes:
  - **X ≤ X**
  - **X ≤ Y** if **X** inherits from **Y**
  - **X ≤ Z** if **X ≤ Y** and **Y ≤ Z**

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- Both rules are ***sound***
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- But more programs typecheck with this new rule
  - It is more ***complete***

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  - some argue for **dynamic typechecking** instead
    - “compute is cheap, human attention is expensive”
  - others argue for **more expressive** static typechecking instead
    - however, more expressive type systems are **more complex**

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> Think of ***least upper bound*** as "nearest common ancestor in the type hierarchy", for now

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  - so, Cool’s soundness theorem is:

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- In

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- And instance of **A** only has method "a()"

Subtyping Example

- Consider the following Cool classes:

  class A { a() : Int { 0 }; }
  class B inherits A { b() : Int { 1 }; }

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- An instance of **B** has methods "a()" and "b()"
- And instance of **A** only has method "a()"
  - A type error will occur if we try to invoke method "b()" on an object with dynamic type A
  - But the static type system will **forbid** such an invocation!

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- Now consider a hypothetical **wrong** let rule:

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  let x : Int <- 0 in x + 1

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- The following program does ***not*** typecheck:

  let x : Int <- 0 in x + 1
- Why not? **Typing environment** hasn't been updated!

Examples of Wrong Let Rule (2)

- Now consider another hypothetical **wrong** let rule:

$$\frac{\Gamma \vdash e_0 : T \qquad T_0 \leq T \qquad \Gamma[T_0/x] \vdash e_1 : T_1}{\Gamma \vdash \textbf{let } x : T_0 \textbf{ <- } e_0 \textbf{ in } e_1 : T_1} \; \textbf{[Let-Init]}$$

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- How is this one different from the correct rule?
- The following ***bad program (!)*** is well-typed:

  let x : B <- new A in x.b()

- Why is this program bad?

Examples of Wrong Let Rule (3)

- Now consider another hypothetical **wrong** let rule:

$$\frac{\Gamma \vdash e_0 : T \qquad T \leq T_0 \qquad \Gamma[T/x] \vdash e_1 : T_1}{\Gamma \vdash \textbf{let } x : T_0 \textbf{ <- } e_0 \textbf{ in } e_1 : T_1} \quad \textbf{[Let-Init]}$$

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- How is this one different from the correct rule?
- This "good" program is not well-typed:

  let x : A <- new B in { … x <- new A ; x.a(); }

- Why isn't this program well-typed?

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Next time we'll cover some **even-more-complex rules** than let:

- Typechecking method dispatch
- Typechecking with SELF_TYPE in Cool

Course Announcements

- Don’t put off starting PA2!
  - This part of the semester may feel like a lull, but that feeling isn’t accurate!