Operational Semantics, Part 2

Martin Kellogg

Agenda

- Review: basics of operational semantics
- Operational semantics of Cool
- (if time): introduction to static analysis
  - further if time: get into abstract interpretation

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- Meaning depends on **context**: a *variable environment* will map variables to memory locations and a ***store*** will map memory locations to values.
  - **environment**: names -> (abstract) locations
  - **store**: (abstract) locations -> values
- We will specify Cool's semantics via **logical rules of inference** that specify how to compute the "next step" in the program

Review: Operational Rules of Cool

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- read as:
  - Given **so**, the current value of the `self` object;
  - and **E**, the current variable environment;
  - and **S**, the current store;
  - and if the evaluation of **e** ***terminates***, then
  - the returned value is **v**
  - and the new store is **S'**

Review: Operational Semantics for Base Values

$$\frac{}{so, E, S \vdash \texttt{true} : \mathrm{Bool(true)}, S}$$

$$\frac{}{so, E, S \vdash \texttt{false} : \mathrm{Bool(false)}, S}$$

$$\frac{i \text{ is any integer literal}}{so, E, S \vdash \texttt{i} : \mathrm{Int}(i), S}$$

$$\frac{\begin{array}{l} s \text{ is any string literal} \\ n \text{ is the length of } s \end{array}}{so, E, S \vdash \texttt{s} : \mathrm{String}(s, n), S}$$

Review: Operational Semantics for Variables

$$\frac{E(\mathtt{id}) = \mathtt{l}_{\mathtt{id}} \qquad S(\mathtt{l}_{\mathtt{id}}) = v}{so, E, S \vdash \mathtt{id} : v, S}$$

- Note the **double lookup** of variables
  - First from name to location (at compile time)
  - Then from location to value (at run time)

Review: Operational Semantics for Assignments

$$\frac{\text{so}, E, S \vdash e : v, S_1 \qquad E(\texttt{id}) = l_{\texttt{id}} \qquad S_2 = S_1[v/l_{\texttt{id}}]}{\text{so}, E, S \vdash \texttt{id <- } e : v, S_2}$$

- A three-step process:
  - Evaluate the right-hand side to get a value v *and* a new store S_1
  - Fetch the location of the assigned variable
  - The result is the value v and an updated store S_2

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- The result of evaluating e_1 is a boolean object
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  - the type rules ensure this
  - there is another, similar, rule for Bool(false)

Operational Semantics for Sequences

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1, S_1 \\ so, E, S_1 \vdash e_2 : v_2, S_2 \\ \dots \\ so, E, S_{n-1} \vdash e_n : v_n, S_n \end{array}}{so, E, S \vdash \{\, e_1 \ ; \ \dots \ ; \ e_n ; \,\} : v_n, S_n}$$

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- Again, the threading of the store expresses the intended execution sequence

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- Only the last value is used

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- Again, the threading of the store expresses the intended execution sequence
- Only the last value is used
- But, all side-effects are collected (how?)

Operational Semantics for While (1)

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$$\frac{so, E, S \vdash e_1 : \text{Bool(false)}, S_1}{so, E, S \vdash \texttt{while}\ e_1\ \texttt{loop}\ e_2\ \texttt{pool} : \text{void}, S_1}$$

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- If e_1 evaluates to Bool(false), then the loop terminates immediately
  - with the side-effects from the evaluation of e_1
  - and with the (arbitrary) result of void
- The type rules ensure that e_1 evaluates to a boolean object

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In-class exercise: given this rule for a **false** loop guard, what do you think the rule for a **true** loop guard looks like?

- In groups of 2 or 3, write down a rule.
- I will collect these; put your UCIDs/emails on it (mostly graded on completion)

(for those reading later online, this is the answer to the in-class exercise on the previous slide)

Operational Semantics for While (2)

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : \text{Bool(true)}, S_1 \\ so, E, S_1 \vdash e_2 : v, S_2 \\ so, E, S_2 \vdash \texttt{while}\ e_1\ \texttt{loop}\ e_2\ \text{pool} : \text{void}, S_3 \end{array}}{so, E, S \vdash \texttt{while}\ e_1\ \texttt{loop}\ e_2\ \text{pool} : \text{void}, S_3}$$

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- Note the sequencing (S -> S_1 -> S_2 -> S_3)

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- Note the sequencing ($S \rightarrow S_1 \rightarrow S_2 \rightarrow S_3$)
- Note how looping is expressed (**recursively!**)
  - Evaluation of a `while` loop is expressed in terms of evaluating a `while` loop in another state

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- Note the sequencing (S -> S_1 -> S_2 -> S_3)
- Note how looping is expressed (**recursively!**)
    - Evaluation of a `while` loop is expressed in terms of evaluating a `while` loop in another state
- The result of evaluating e_2 is discarded; only the side-effects are kept

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$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1, S_1 \\ so, ?, ? \vdash e_2 : v_2, S_2 \end{array}}{so, E, S_1 \vdash \texttt{let}\ id : T\ \texttt{<-}\ e_1\ \texttt{in}\ e_2 : v_2, S_2}$$

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- What is the context in which e_2 should be evaluated?
  - Environment should be like **E** but with a new binding of **id** to a fresh location $\texttt{l}_{\texttt{new}}$
  - Store like S_1 but with $\texttt{l}_{\texttt{new}}$ mapped to v_1

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- This lets(haha) us write the correct let rule:

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1, S_1 \\ \mathtt{l}_{\mathtt{new}} = \mathtt{newloc}(S_1) \\ so, E[\mathtt{l}_{\mathtt{new}}/id], S_1[v_1/\mathtt{l}_{\mathtt{new}}] \vdash e_2 : v_2, S_2 \end{array}}{so, E, S_1 \vdash \mathtt{let}\ id : T \mathtt{<-}\ e_1\ \mathrm{in}\ e_2 : v_2, S_2}$$

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- This may initially seem tricky
  - How could that possibly work?
  - What's going on here?
- Once you’ve studied them a bit, hopefully you’ll agree they’re actually **quite elegant**
  - But they will probably not seem that way at first

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  - Evaluate the initializers and set the resulting attribute values
  - Return the newly allocated object

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  - D_{String} = String(0, “”)
  - D_A = void   for all other classes **A**

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This is exactly the *class map* from PA2!

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$$\frac{\begin{array}{c} T_0 = \text{if } T = \textbf{SELF\_TYPE} \text{ and } \textbf{so} = X(\ldots) \text{ then } X \text{ else } T \\ \text{class}(T_0) = (a_1 : T_1 \text{ <- } e_1, \ldots, a_n : T_n \text{ <- } e_n) \\ \forall\, i \in [1 \ldots n],\ \mathtt{l}_i = \mathtt{newloc}(S) \\ v = T_0(a_1 = \mathtt{l}_1, \ldots, a_n = \mathtt{l}_n) \\ S_1 = S[D_{T1}/\mathtt{l}_1, \ldots, D_{Tn}/\mathtt{l}_n] \\ E' = [a_1 : \mathtt{l}_1, \ldots, a_n : \mathtt{l}_n] \\ \textbf{so}, E', S_1 \vdash \{a_1 \text{ <- } e_1 ; \ldots ; a_n \text{ <- } e_n ;\} : v_n, S_2 \end{array}}{\textbf{so}, E, S \vdash \mathtt{new}\ T : v, S_2}$$

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if the desired type is **SELF_TYPE**, use the **so** object; otherwise use the type named in the expression (**T**)

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$$v = T_0(a_1 = l_1, \ldots, a_n = l_n)$$

$$S_1 = S[D_{T1}/l_1, \ldots, D_{Tn}/l_n]$$

$$E' = [a_1 : l_1, \ldots, a_n : l_n]$$

$$\frac{\textbf{so}, E', S_1 \vdash \{a_1 \leftarrow e_1 ; \ \ldots \ ; a_n \leftarrow e_n ;\} : v_n, S_2}{\textbf{so}, E, S \vdash \texttt{new } T : v, S_2}$$

fetch the template for the class to instantiate

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make space for each of its attributes (now, we've *allocated* the new object)

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create a new value for the newly-created object; make each attribute point to the appropriate new location (this step is the start of *initialization*)

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$$
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T_0 = \text{if } T = \textbf{SELF\_TYPE} \text{ and } \textbf{so} = X(\ldots) \text{ then } X \text{ else } T \\
\text{class}(T_0) = (a_1 : T_1 \texttt{<-} e_1, \ldots, a_n : T_n \texttt{<-} e_n) \\
\forall\, i \in [1 \ldots n],\ \texttt{l}_i = \texttt{newloc}(S) \\
v = T_0(a_1 = \texttt{l}_1, \ldots, a_n = \texttt{l}_n) \\
S_1 = S[D_{T1}/\texttt{l}_1, \ldots, D_{Tn}/\texttt{l}_n] \\
E' = [a_1 : \texttt{l}_1, \ldots, a_n : \texttt{l}_n] \\
\textbf{so}, E', S_1 \vdash \{a_1 \texttt{<-} e_1 ; \ldots ; a_n \texttt{<-} e_n ;\} : v_n, S_2
\end{array}}{\textbf{so}, E, S \vdash \texttt{new } T : v, S_2}
$$

create a new store with each new attribute location set to the **default value** for its type

Operational Semantics for new

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create a new environment with only the attributes **in-scope** (in which to evaluate the initializers)

Operational Semantics for new

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evaluate all of the initializers, keeping the side-effects in S_2

Trivia Break: Real World Languages

English is the single-most widely-spoken and only official language in this West African country, which, with over 230 million people, is the most populous country in Africa (and its former capital, Lagos, is one of Africa's largest cities). The country's linguistic diversity is a microcosm of Africa as a whole, with significant numbers of native speakers of languages from the three major African language families: Afroasiatic, Nilo-Saharan and Niger-Congo.

Name the country and any one language of African origin that is spoken there by at least 2 million people.

Trivia Break: Math

This Austrian mathematician moved to New Jersey in a rather circuitous way: after the Anschluss in 1938, the Nazis found him - previously a lecturer at the University of Vienna - fit for conscription. He fled across the Soviet Union, sailed to Japan and then on to San Francisco, and then traveled across the US to take up a position at the Institute for Advanced Study (IAS) in Princeton. Toward the end of his own life, fellow IAS researcher Albert Einstein confided that his "own work no longer meant much, that he came to the Institute merely ... to have the privilege of walking home with [him]". Though his work spanned several areas of mathematics, philosophy, and logic, he is most famous for his Incompleteness Theorem.

Operational Semantics of Method Dispatch

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  - Initialize the locations with the actual arguments
  - Set self to the target object and evaluate `f`'s body

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This is exactly the ***implementation map*** from PA2!

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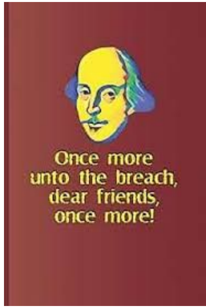
Operational Semantics for Method Dispatch



Operational Semantics for Method Dispatch

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\ldots \qquad so, E, S_{n-1} \vdash e_n : v_n, S_n \\
so, E, S_n \vdash e_0 : v_0, S_{n+1} \\
v_0 = X(a_1 = l_1, \ldots, a_m = l_m) \\
imp(X, f) = (x_1, \ldots, x_n, e_{body}) \\
\forall\, i \in [1 \ldots n],\ l_{xi} = \mathtt{newloc}(S_{n+1}) \\
E' = [x_1 : l_{x1}, \ldots, x_n : l_{xn}, a_1 : l_1, \ldots, a_m : l_m] \\
S_{n+2} = S_{n+1}[v_1/l_{x1}, \ldots, v_n/l_{xn}] \\
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\end{array}
}{
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}
$$

evaluate all of the arguments

Operational Semantics for Method Dispatch

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evaluate the receiver object (= object on which method is called)

Operational Semantics for Method Dispatch

$$\frac{\begin{array}{ll} so, E, S \vdash e_1 : v_1, S_1 & so, E, S_1 \vdash e_2 : v_2, S_2 \\ \ldots & so, E, S_{n-1} \vdash e_n : v_n, S_n \\ so, E, S_n \vdash e_0 : v_0, S_{n+1} & \\ v_0 = X(a_1 = l_1, \ldots, a_m = l_m) & \\ imp(X, f) = (x_1, \ldots, x_n, e_{body}) & \\ \forall\, i \in [1 \ldots n],\ l_{xi} = newloc(S_{n+1}) & \\ E' = [x_1 : l_{x1}, \ldots, x_n : l_{xn}, a_1 : l_1, \ldots, a_m : l_m] & \\ S_{n+2} = S_{n+1}[v_1/l_{x1}, \ldots, v_n/l_{xn}] & \\ v_0, E', S_{n+2} \vdash e_{body} : v, S_{n+3} & \end{array}}{so, E, S \vdash e_0.f(e_1, \ldots, e_n) : v, S_{n+3}}$$

find the receiver's type and attributes

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find formals and body

Operational Semantics for Method Dispatch

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call by reference or by value?

Aside: Call by Reference vs Call by Value

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- Which does C support?

Operational Semantics for Method Dispatch

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call by reference, only allocate space for copies of the pointers

Operational Semantics for Method Dispatch

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new environment, with the formals and the attributes of the receiver in-scope

Operational Semantics for Method Dispatch

so, E, S ⊢ … S_2

…

… , S_n

so, E, S_n

v_0 = X(a_1

imp(X, f) = (x_1, …, x_n, e_{body})

Do you think the **order matters** here? What could go wrong if the formals were after the attributes?

$\forall\, i \in [1 \ldots n]$, $\mathtt{l}_{xi}$ = newloc(S_{n+1})

E' = [x_1 : $\mathtt{l}_{x1}$, … , x_n : $\mathtt{l}_{xn}$, a_1 : $\mathtt{l}_1$, … , a_m : $\mathtt{l}_m$]

new environment, with the formals and the attributes of the receiver in-scope

S_{n+2} = S_{n+1}[v_1/$\mathtt{l}_{x1}$, … , v_n/$\mathtt{l}_{xn}$]

v_0, E', S_{n+2} ⊢ e_{body} : v, S_{n+3}

---

so, E, S ⊢ e_0.f(e_1, …, e_n) : v, S_{n+3}

Operational Semantics for Method Dispatch

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new store with formals pointing to the actual arguments

Operational Semantics for Method Dispatch

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finally, evaluate the body

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- The body of the method is invoked with:
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  - $\mathbf{S}_{\mathbf{n+2}}$ like the caller's except with actual arguments bound to the locations allocated for formals
- The notion of the **activation frame** is implicit
  - New locations are allocated for actual arguments
- The semantics of **static dispatch** is similar except the implementation of `f` is taken from the specified class

Run-time Errors

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$$v_0 = X(a_1 = \mathtt{l}_1, \ldots, a_m = \mathtt{l}_m)$$

$$imp(X, \mathtt{f}) = (x_1, \ldots, x_n, e_{body})$$

$$\ldots$$

- What happens if imp(X, $\mathtt{f}$) is not defined?

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$$\ldots$$

- What happens if $imp(X, \mathtt{f})$ is not defined?
  - It **cannot be**! Type safety theorem guarantees it :)

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  - heap overflow
- In such cases the execution must **abort gracefully**

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  - division by zero
  - substring out of range
  - heap overflow
- In such cases the execution must **abort gracefully**
  - i.e., with an error message rather than a segfault

Run-time Errors

- There are some run-time errors that the typechecker **does not try** to prevent (but it could - we'll get to it in a few minutes)
  - dispatching on void
  - division by zero
  - substring out of range
  - heap overflow
- In such cases the execution must **abort gracefully**
  - i.e., with an error message rather than a segfault
  - implication: you must generate code in PA3 that **checks for run-time errors**!

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- Operational rules contain a lot of details
  - Read them **carefully**!
- Most languages **do not** have a well-specified operational semantics :(
- When **portability** is important, an operational semantics is essential
  - But typically not using the exact notation we used for Cool

Agenda

- Review: basics of operational semantics
- Operational semantics of Cool
- **(if time): introduction to static analysis**
    - further if time: get into abstract interpretation

Motivation: many defects are hard to test for

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This is especially true for certain kinds of hard-to-test-for defects that might not be apparent even if you do exercise them, such as **resource leaks**

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  - API Protocols: device drivers, GUI frameworks
  - Exceptions: arithmetic, library, user-defined
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There are **rules** for doing each of these things **correctly**, and a static analysis can automate those rules.

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We have already encountered one kind of static analysis in this class: **type systems**. Type systems aren't special - they are just a very common static analysis.

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This means that we don't need an **external model** of what the computer does!
(Since your compiler faithfully implements the OpSem, right?)

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- **Unsound**: results may not generalize to future executions
  - Describes execution environment or test suite

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Static Analysis Properties

- **Slow** to analyze large models of state, so use abstraction
- **Conservative**: account for abstracted-away state
- **Sound**: (weak) properties are guaranteed to be true
    - Some static analyses are not sound, but static analyses ***can*** be made sound

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Analogous Analyses

- **Any** analysis problem can be solved with **either** a static or a dynamic analysis
  - e.g., consider **type safety**: no memory corruption or operations on wrong types of values
    - Static type-checking (e.g., Java, Cool)
    - Dynamic type-checking (e.g., Python)
- This insight gives us a kind of "***PL incompleteness theorem***": either you can know something precisely about one execution (via dynamic analysis) or imprecisely about every execution (via static analysis)

Static vs Dynamic Analyses

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- Static analysis focuses on a **subset of data structures**
  - more precise for data or control described by the abstraction
  - typically **conservative** / **pessimistic** elsewhere
    - i.e., assume that unmodeled state is unsafe

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  - including **type systems** (which we've already discussed)
  - formally, abstract interpretation is expressive enough that you can describe *any* static analysis using it
    - that said, you probably don't want to
    - ask me more about Patrick Cousot's work in OH

Course Announcements

- **PA2c2** due today
  - if you haven't started yet, you almost certainly won't finish in time (come talk to me about it)
- I'll hold **two short OH today** for those who want to see a test case before PA2c2:
  - right after class (11:25-11:55am)
  - 4:30-5pm
- **PA2 (full)** is due next Monday (one week from today!)

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