Run-time Organization and Operational Semantics

Martin Kellogg

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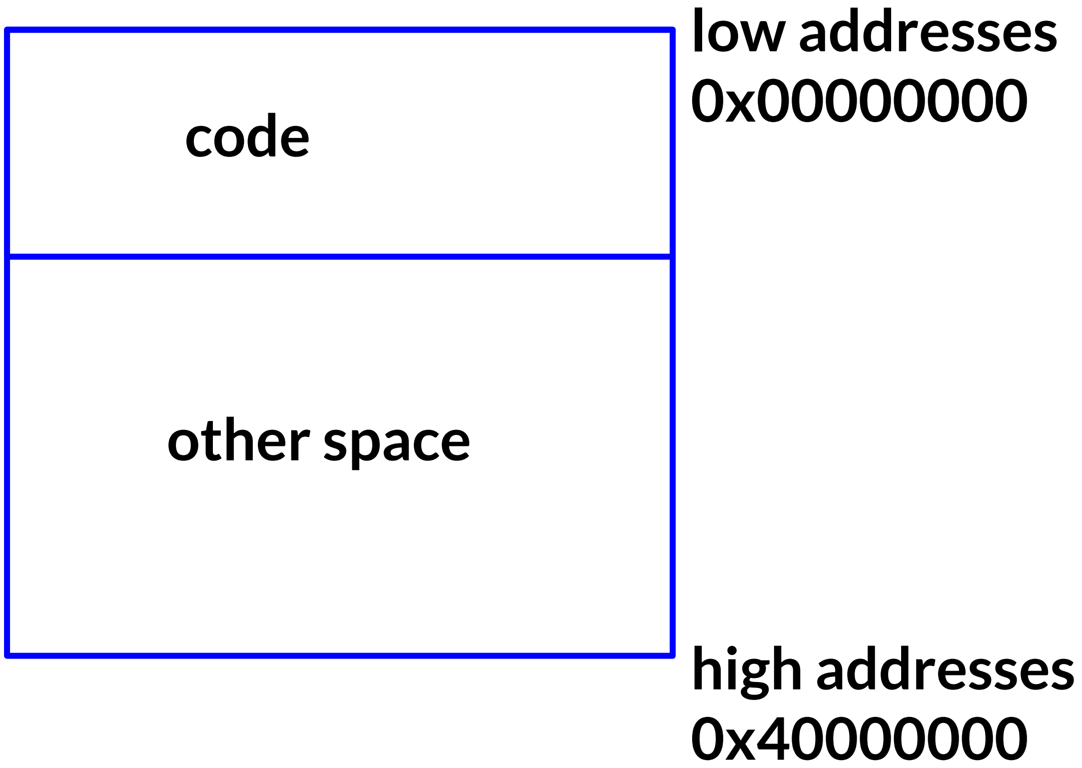
Review: Virtual Memory Picture

a program's virtual memory:



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  - when these conflict, **correctness** comes first

Review: Assumptions About Execution

- **Assumption (1):** Execution is **sequential**; control moves from one point in a program to another in a well-defined order

(note that we know these assumptions are false in real life! See last lecture for details...)

Review: Assumptions About Execution

- **Assumption (1):** Execution is **sequential**; control moves from one point in a program to another in a well-defined order
- **Assumption (2):** When a procedure is called, control eventually returns to the point **immediately after the call**

(note that we know these assumptions are false in real life! See last lecture for details...)

Review: Activation Trees

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- Example ->

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class Main {
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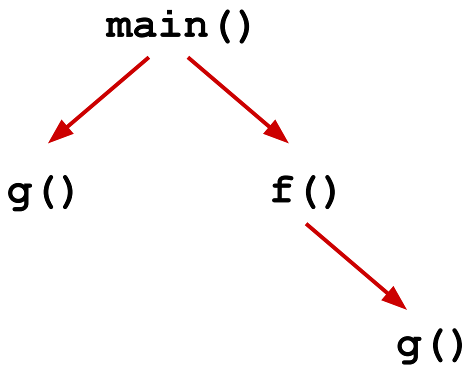

Activation Tree Notes

- The activation tree **depends on run-time behavior**
  - The activation tree may be different for every program input
- Since activations are properly nested, a **stack** can track currently active procedures
  - This is the ***call stack***

Activation Tree Example Revisited

- Let's track activations with a stack on the example from before:

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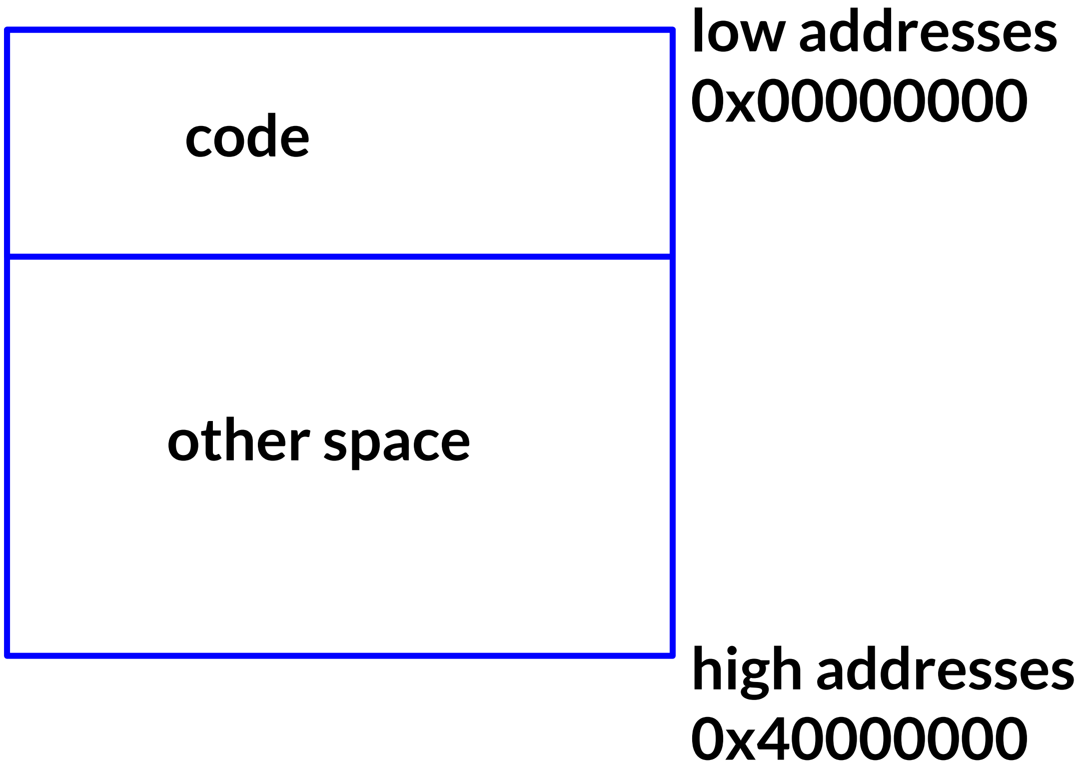

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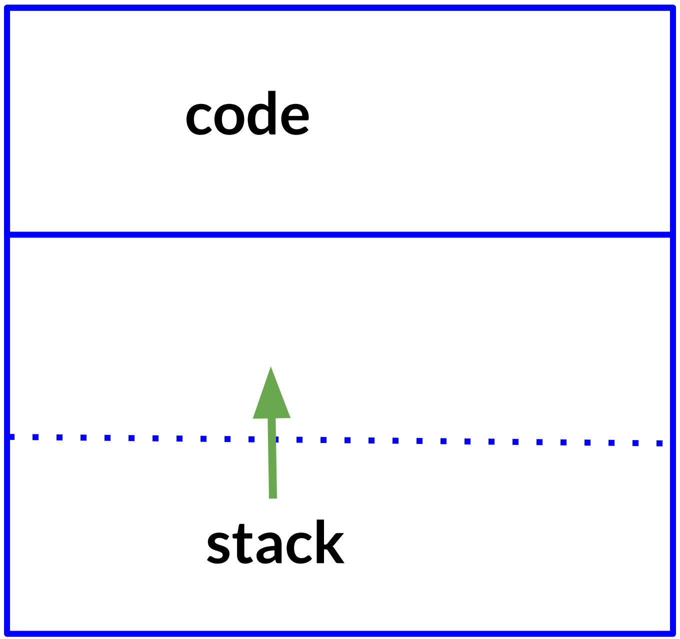

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  - Register and program counter contents
- Other temporary values

Revisiting An Example

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class Main {
  g() : Int { 1 };
  f(x : Int) : Int {
    if x = 0
      then g()
      else f(x - 1) (**)
    fi
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    f(3); (*) }};
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AR for **f**:

return address
control link
argument
space for result

Revisiting An Example: Stack after 2 Calls to f()

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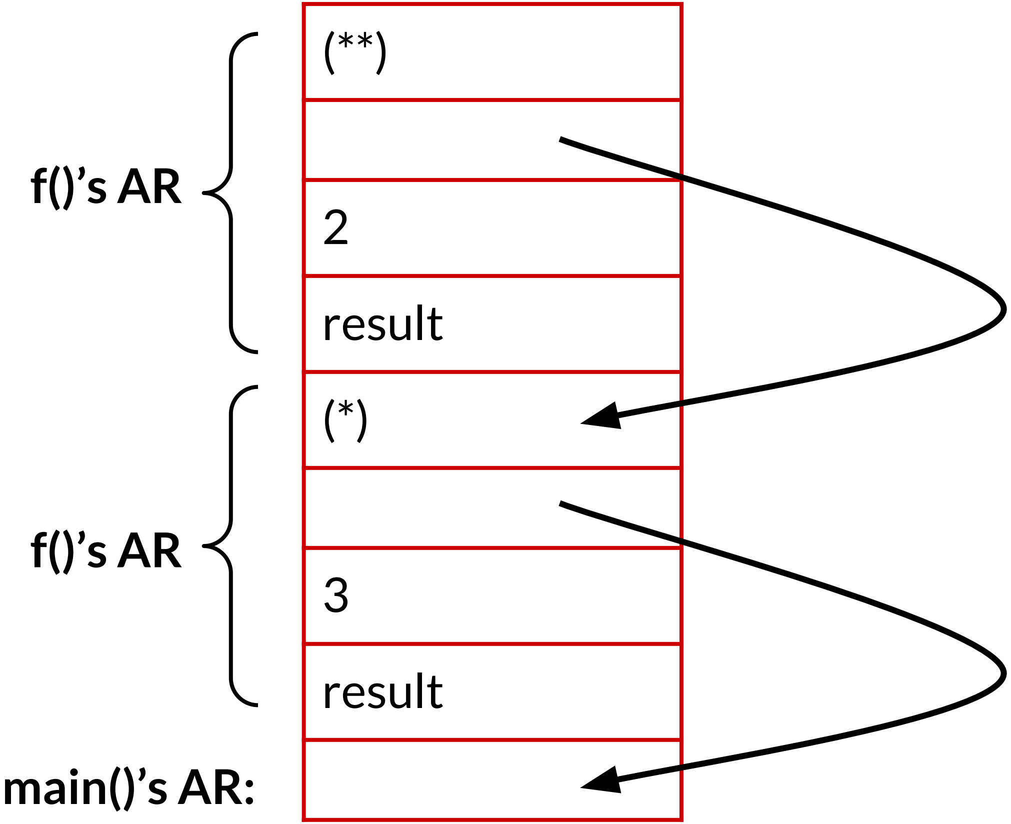

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- **main()** has no argument or local variables and its result is "never" used; its AR is uninteresting
- (*) and (**) are return addresses of the invocations of f
  - The ***return address*** is where execution resumes after a procedure call finishes
- This is only one of many possible AR designs
  - Would also work for C, Pascal, FORTRAN, etc.

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Thus, the AR layout and the compiler must be ***designed together****!*

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- Real compilers hold as much of the frame as possible in ***registers***
  - Especially method result and arguments
- Why?

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- Depending on the language, there may be other statically allocated values

Memory Layout with Static Data

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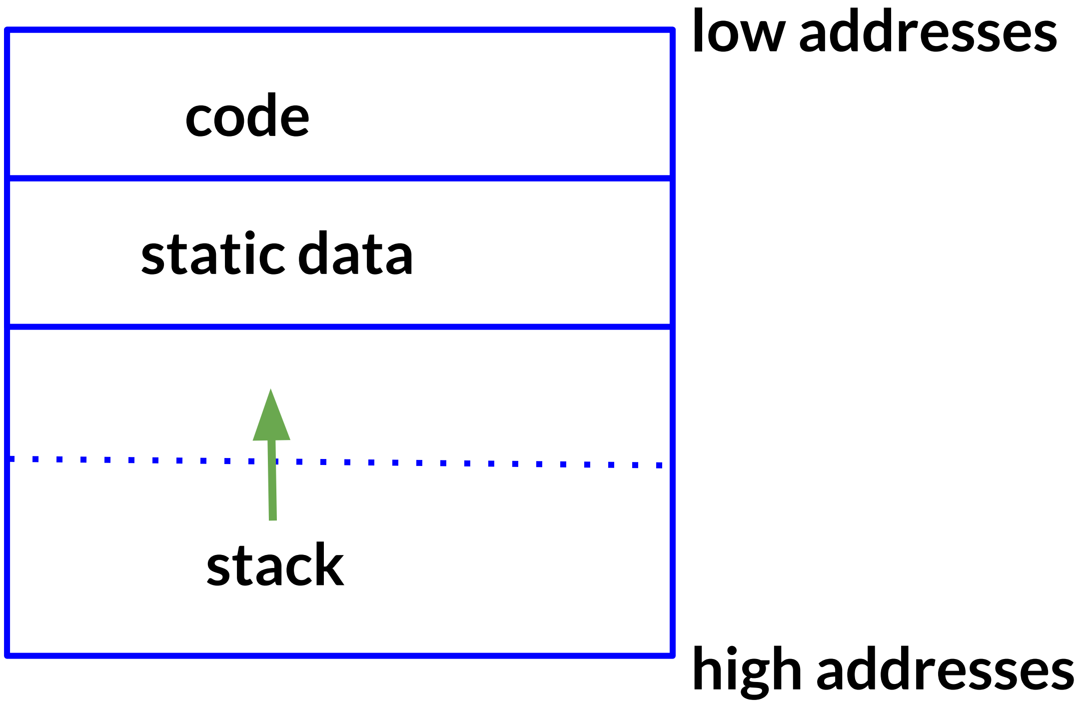

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- Languages with dynamically-allocated data (such as Cool!) use a ***heap*** to store such dynamic data

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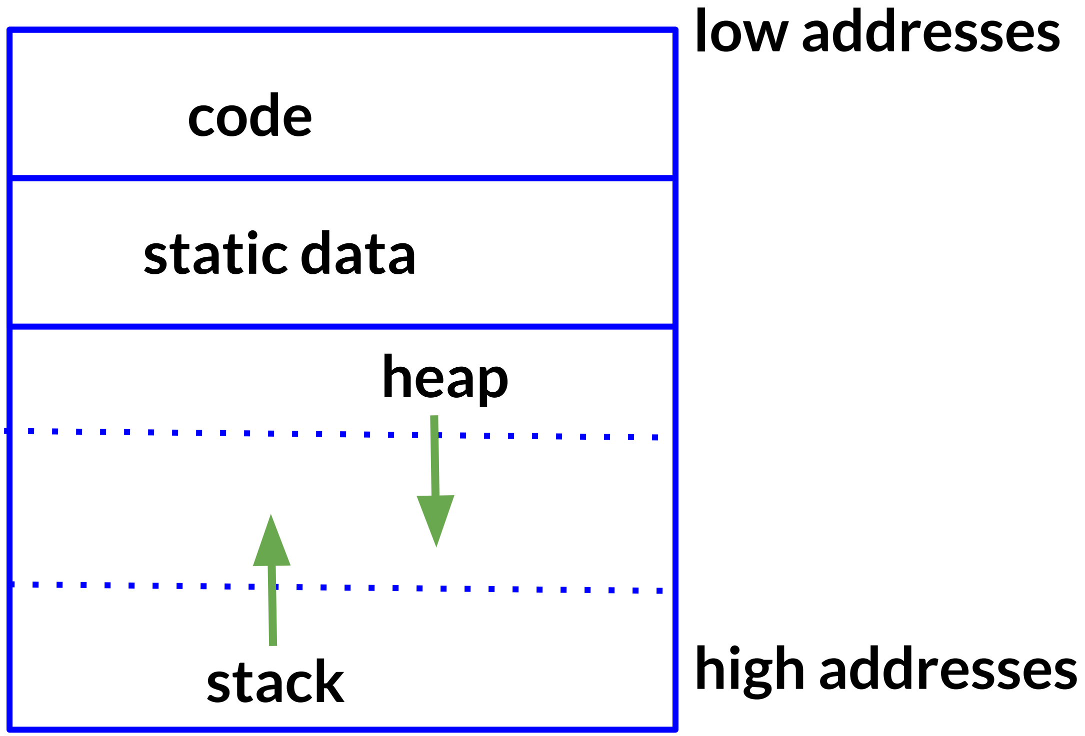

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- You'll need to use and manage an explicit heap (introduced today and covered in more detail in later lectures). A heap maps addresses (i.e., integers) to values.

Trivia Break: Computer Science

This living British computer scientist won the Turing award in 1980. Born in 1934 to British parents living in Sri Lanka, he studied classics as an undergraduate at Oxford and only began programming in graduate school. He spent time while a graduate student at Moscow State University in the Soviet Union as part of an exchange program, where he studied under Andrey Kolmogorov. While his work is foundational in program verification, he is best known for developing the quicksort algorithm.

Trivia Break: Gaming

This game was originally designed by Gary Gygax and Dave Arneson in the early 1970s, as a derivative of the miniature wargame *Chainmail*. The main difference between it and its predecessor wargames is that this game allows players to make a specific character to control, rather than forcing players to control an entire military formation. Its publication is commonly recognized as the beginning of the modern tabletop role-playing genre, and it has deeply influenced both modern tabletop and video games. The game is currently in its 5th edition, and is published by Wizards of the Coast.

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    - I warned you there was a relationship to type systems…

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(Rest of) today's plan:

- Motivation
- Notation
- The Rules

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  - The **grammar** (syntactic analysis/parsing)
  - The **type rules** (semantic analysis)
  - The ***evaluation rules*** (interpretation)
    - also: staged hints for compilation!

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- Why isn’t this description good enough?

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- But assembly-language descriptions of language implementation have too many **irrelevant details**
  - Which way the stack grows
  - How integers are represented on a particular machine

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- We need a **complete** but **not overly restrictive** specification

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- the **abstract machine** is a **mathematical representation of computation** (where have we seen one of those before?)
- this semantics is most useful for **specifying an implementation**
  - and it's what we'll use for Cool

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    - e.g., that quicksort returns a sorted array
  - The foundation for many **program verification** tools
    - Ask me about Hoare logic in office hours to learn more!

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- We try something similar for evaluation:

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Example Operational Semantics Rule

$$\frac{\begin{array}{c}\textbf{context} \vdash e_1 : 5 \\ \textbf{context} \vdash e_2 : 7\end{array}}{\textbf{context} \vdash e_1 + e_2 : 12}$$

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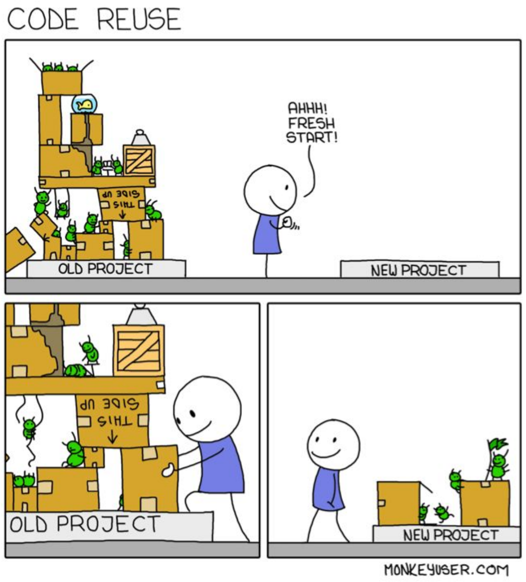
- In general the result of evaluating an expression **depends on** the result of evaluating its subexpressions
- The logical rules specify **everything** that is needed to evaluate an expression

Aside: Complexity of OpSem Rules

- The operational semantics inference rules for Cool will become **complicated**
  - i.e., many hypotheses



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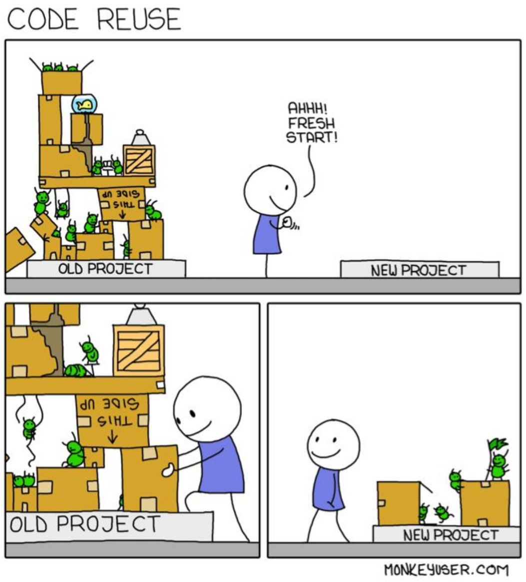

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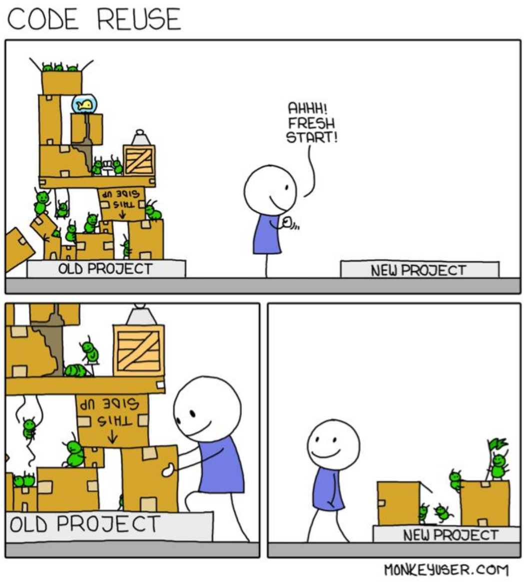

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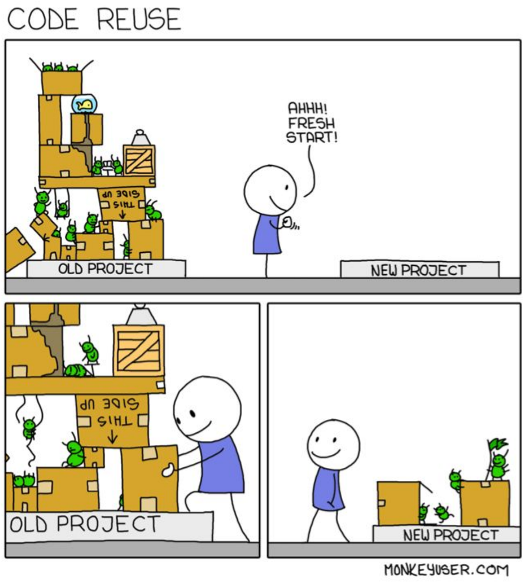

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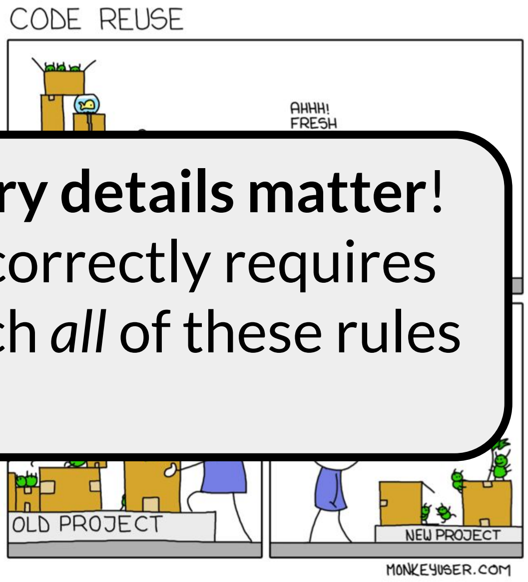

These **theory details matter!** Compiling correctly requires you to match *all* of these rules exactly…

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> Why do you think we need to **separate** the environment and the store? (Hint: which is static? Dynamic?)

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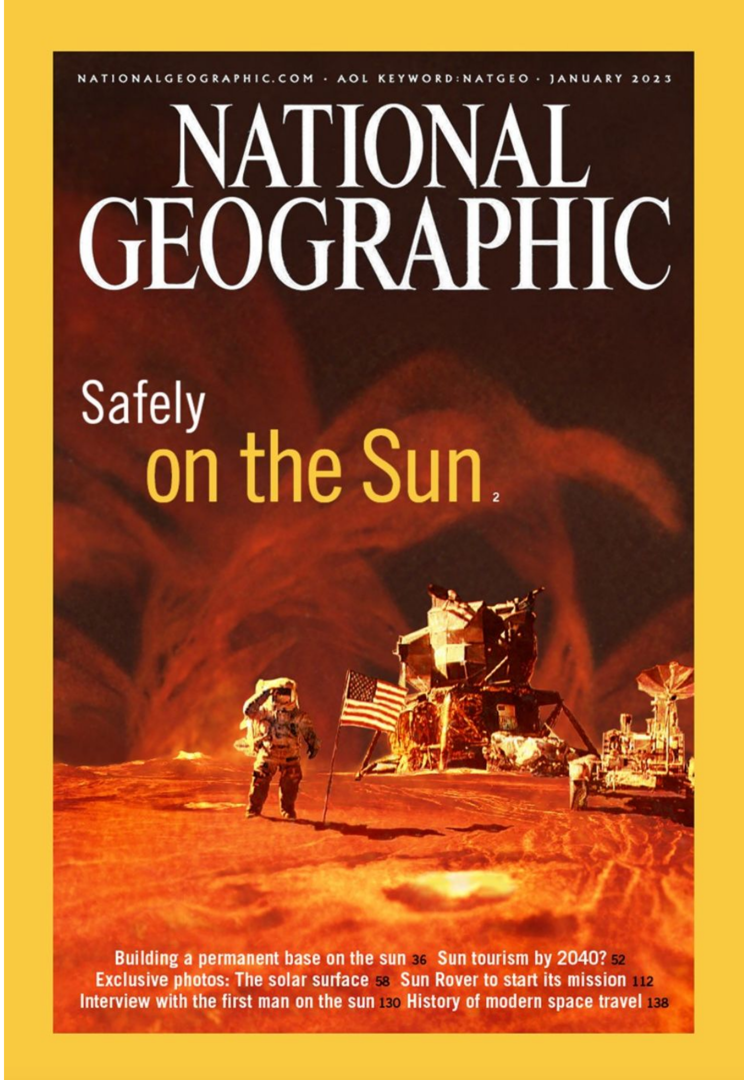
- To lookup a variable a in environment E we write E(a)

Lost?

- Environments may seem hostile and unforgiving



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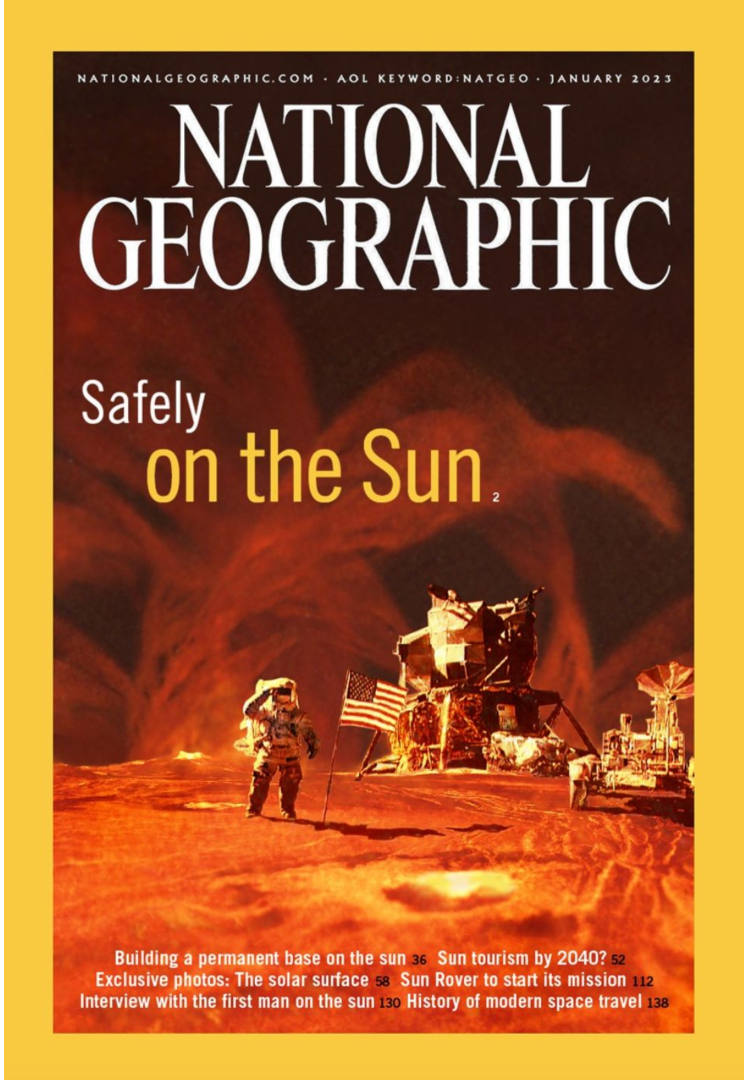

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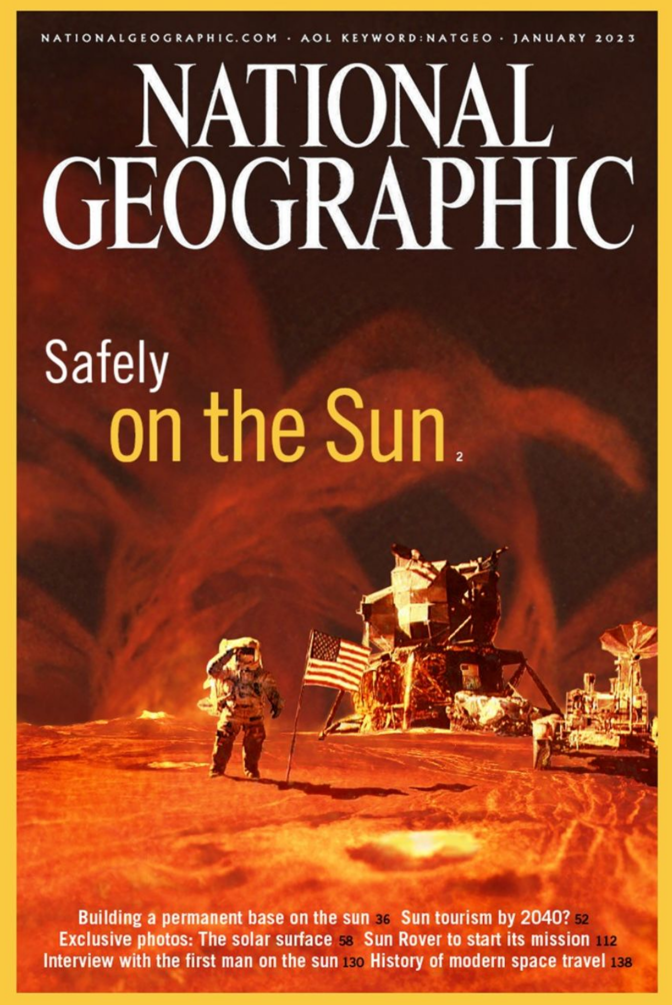

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  - This denotes a new store S' such that $S'(l_1) = 23$ and $S'(l) = S(l)$ if $l \neq l_1$

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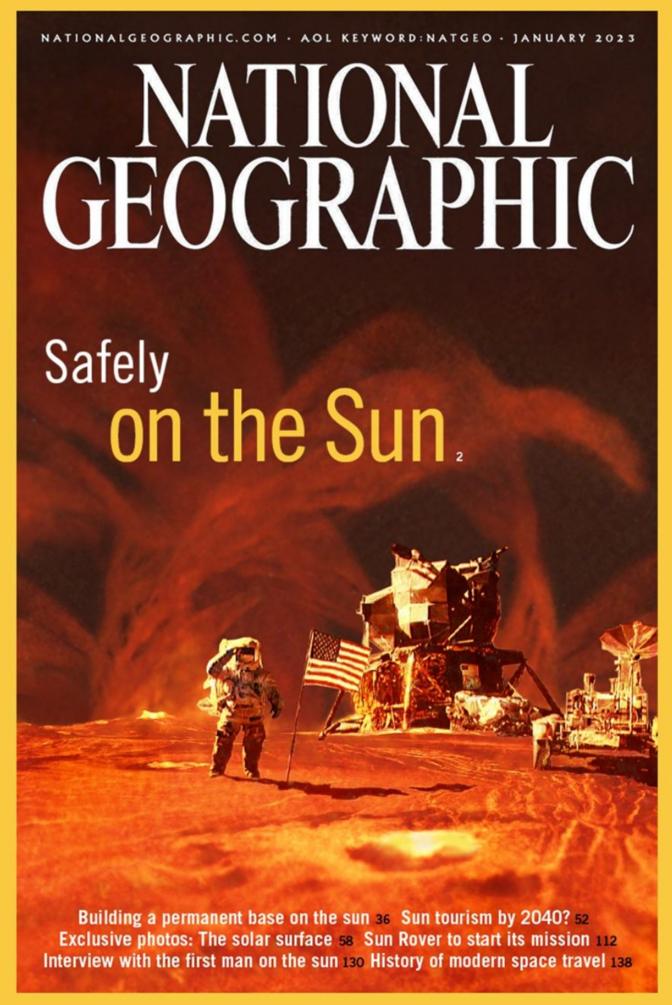

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  - Concrete implementations might use `NULL` here

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- We’ll define one rule for each kind of expression

Operational Semantics for Base Values

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- note: no side-effects in these cases (i.e., the store doesn't change)

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  - Fetch the location of the assigned variable

Operational Semantics for Assignments

$$\frac{\begin{array}{c} so, E, S \vdash e : v, S_1 \\ E(\mathtt{id}) = \mathtt{l}_{\mathtt{id}} \qquad S_2 = S_1[v/\mathtt{l}_{\mathtt{id}}] \end{array}}{so, E, S \vdash \mathtt{id} \texttt{ <- } e : v, S_2}$$

- A three-step process:
  - Evaluate the right-hand side to get a value v *and* a new store S_1
  - Fetch the location of the assigned variable
  - The result is the value v and an updated store S_2

Operational Semantics for Assignments

$$\frac{\begin{array}{c}\text{so}, E, S \vdash e : v, S_1 \\ E(\texttt{id}) = \texttt{l}_{\texttt{id}} \qquad S_2 = S_1[v/\texttt{l}_{\texttt{id}}]\end{array}}{\text{so}, E, S \vdash \texttt{id <- } e : v, S_2}$$

- A three-step process:
  - Evaluate the right-hand side to get a value **v** *and* a new store S_1
  - Fetch the location of the assigned variable
  - The result is the value **v** and an updated store S_2
- The environment doesn't change

Course Announcements

- **PA2c2** still due next Monday
  - requires typechecking + semantic analysis of everything but expressions
  - if you haven’t started yet, I’m now **very worried** for you