Local Optimizations

Martin Kellogg

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- We recently fixed a bug in the reference compiler’s x86-64 module. Only use Cool **version 1.39** for compiling to x86.

Agenda

- Introduction to optimization
    - Goals
    - Taxonomy
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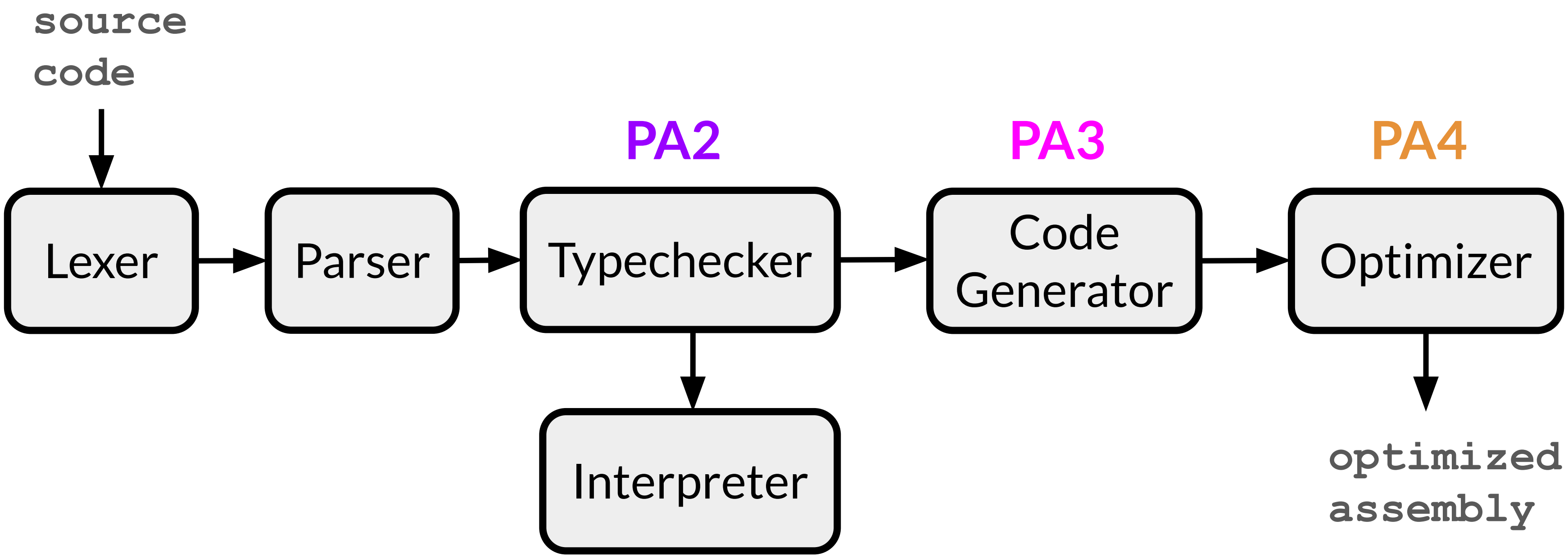

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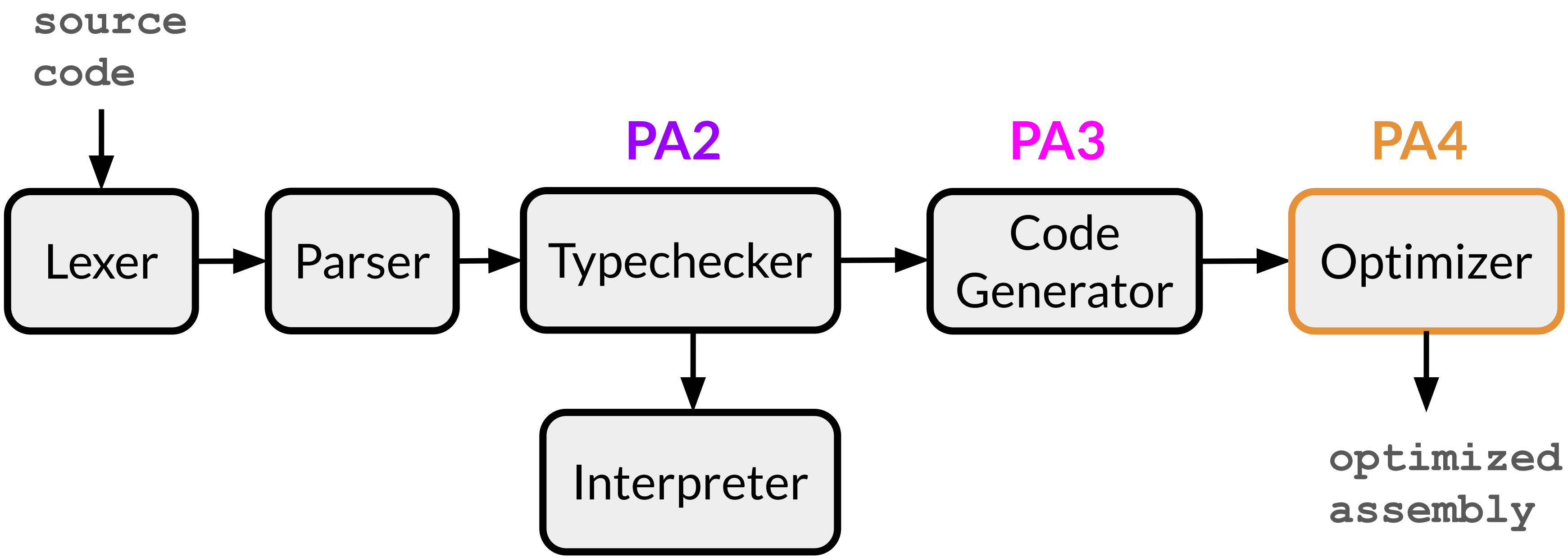

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  - “optimization” is a bad name: there’s nothing “optimal” about the result
  - “**program improvement**” is probably more apt

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        - common ways to define profit: fewer clock cycles, smaller binary size, uses less battery, etc.

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  - you can also use facts from any previous compilation stage

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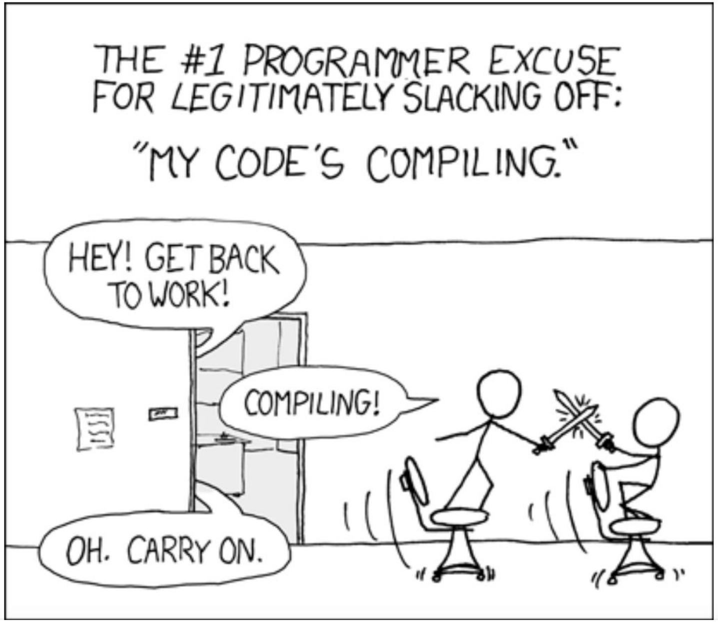

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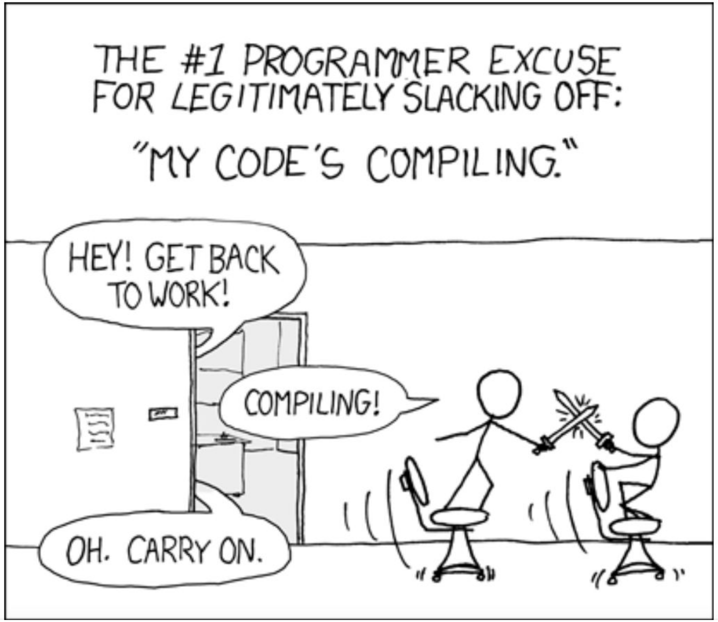

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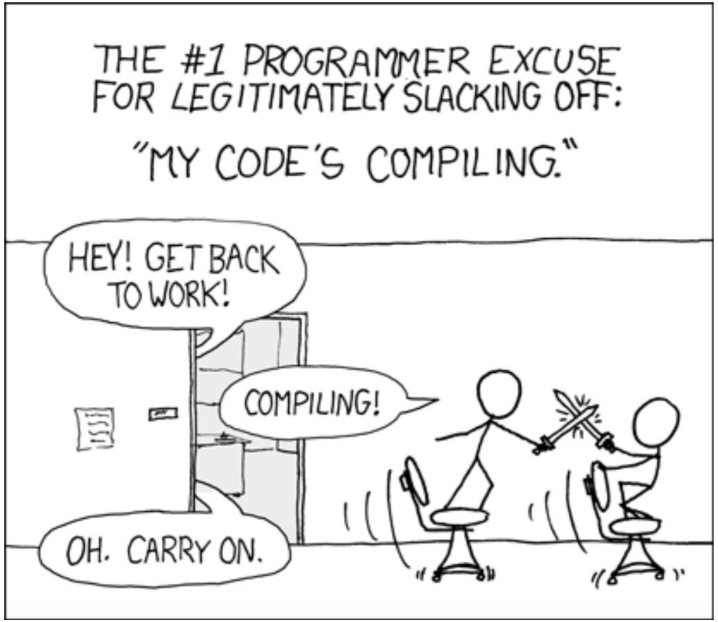

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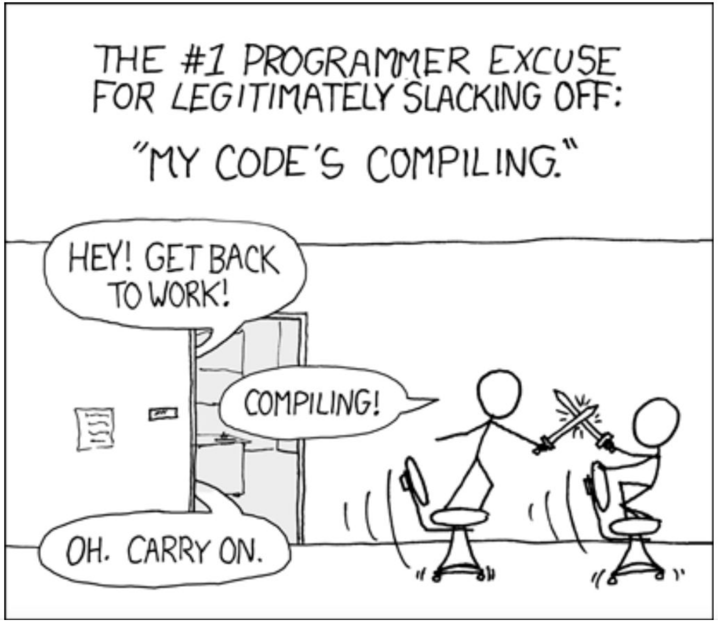

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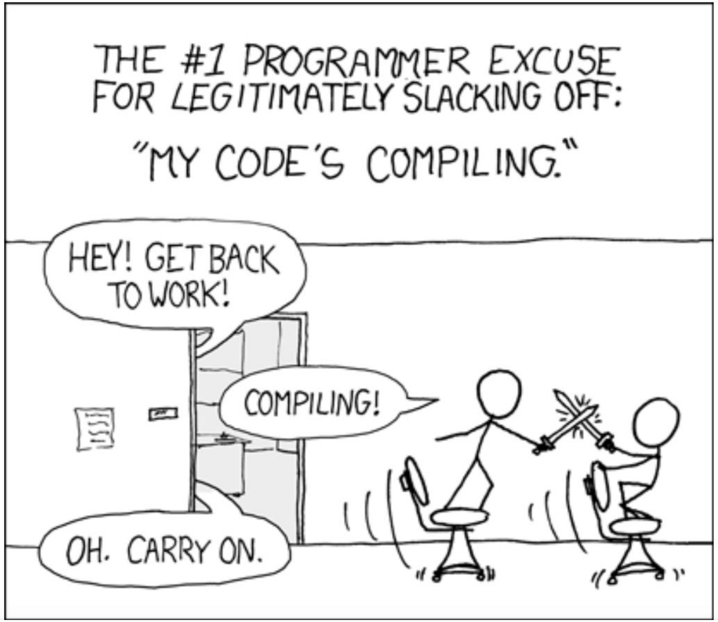
- Our goal: maximum improvement with minimum of cost (no risk!)



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  - Code gen (e.g., your PA3) is initially designed to produce the "most general form" of each language construct
  - Often, a simpler version will work instead
    - e.g., if you know that a pointer is definitely non-void, do you need to emit the code for dispatch on void?

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This list is not exhaustive! For PA4, you have a lot of **creative freedom** to choose what you want to optimize.

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- The ***compiler backend*** is responsible for such decisions
    - e.g., mapping abstract registers to “real” x86-64 registers (***register allocation***)
- Many optimizations are possible at the backend level, too:
    - spill fewer registers
    - select “better” instructions
    - schedule instructions in a way that wastes fewer clock cycles

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  - we'll use **static analysis** to check -> we inherit all problems of static analysis (such as?)

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Today: some **peephole**
+ local optimizations

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- Another example: “jump chains” (of unconditional jumps) can be replaced with a single jump

Peephole Optimizations: More Examples

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subq $8, %rax
movq %r2, 0(%rax)
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One way to do **complex instruction** selection

Trivia Break: Computer Science

This graph algorithm was independently invented by four different computer scientists in the 1950s; it is usually named after two or sometimes three of its inventors. Djikstra's shortest path algorithm can solve most instances of the problem that this algorithm addresses (and is faster). However, the advantage of this algorithm is that it can handle graphs with negative edge weights, including detecting "negative cycles" with infinite profitability (i.e., graphs in which no path is "shortest", because of the negative cycle).

Trivia Break: Medicine

This woman from Roanoke, Virginia died of cervical cancer in 1951 (aged just 31) at Johns Hopkins University in Baltimore. Without her consent, the medical researcher George Otto Gey cultured a line of cells from a tumor biopsy taken while she was being treated for cervical cancer. This cell line was the first "immortalized" human cell line that could be cloned indefinitely, and continues to be used in medical research to this day. Its cells are durable and prolific, and by 2009 more than 60,000 scientific articles had been published about research using the line. Name either the cell line or the woman whose cells were used to create it.

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Local Optimizations

- A simple form of optimizations
  - No need to analyze the whole procedure body
  - Just the **basic block** in question
- Every (production) compiler does these
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- Examples:
  - algebraic simplification
  - constant folding
  - local value numbering
  - …

Local Optimizations: Algebraic Simplification

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x := x * 15     ->
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- (watch out: on some machines `<<` is faster than `*`; but not on all!)

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  - Such basic blocks can be eliminated
- Why would such basic blocks occur?
- Removing unreachable code makes the program smaller
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    - Due to memory cache effects (increased spatial locality)

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- Example (x_1 and a_1 are fresh temporaries):

```
x := a + y
a := x
x := a * x
b := x + a
```

→

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- Intermediate code can be rewritten to be in **single static assignment** form (it's just another IR)
- Example (x_1 and a_1 are fresh temporaries):

```
x  := a + y                x   := a + y
a  := x          →         a1  := x
x  := a * x                x1  := a1 * x
b  := x + a                b   := x1 + a1
```

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- Makes many analyses (and related optimizations) more efficient
- Separates values from memory storage locations
- Complementary to CFG or data dependency graph
    - better for some things, but cannot do everything

SSA vs Functional Programming

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$$
\begin{array}{l}
\texttt{x := a + y} \\
\texttt{a}_1\ \texttt{:= x} \\
\texttt{x}_1\ \texttt{:=a}_1\ \texttt{* x} \\
\texttt{b := x}_1\ \texttt{+ a}_1
\end{array}
\quad \cong \quad
\begin{array}{l}
\texttt{let x = a + y in} \\
\texttt{let a}_1\ \texttt{= x in} \\
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\ldots
\end{array}
$$

SSA: Basic Idea

- Basic Idea: for each original variable *v*, create a new variable v_n at the *n*th definition of the original *v*. Subsequent uses of *v* use v_n until the next definition point. E.g.:

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```
a := x + y
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a := y + b
b := x * 4
a := a + b
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SSA:

$a_1 := x_0 + y_0$
$b_1 := a_1 - 1$
$a_2 := y_0 + b_1$
$b_2 := x_0 * 4$
$a_3 := a_2 + b_2$

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if (...)
   a = x;
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$a_3 = \varphi(a_1, a_2)$
$b_1 = a_3;$

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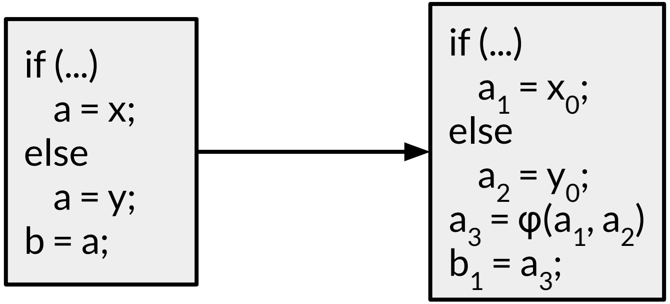

- Solution: introduce a ***φ-function*** (“phi function”)
  - semantics: a_3 is assigned to either a_1 or a_2, depending on which control flow path us used to reach the φ-function

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    - So φ-functions are (only) compile-time bookkeeping

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  - There are non-empty paths from x to z and from y to z
  - These paths have no common nodes other than z

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- Why is SSA form important here?

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- This does not make the program smaller or faster
  - But it might enable other optimizations!
    - Constant folding, dead code elimination
- Again, being in SSA is critical to the proof that this is safe

Example: Copy Propagation + Constant Folding

```
a := 5
x := 2 * a
y := x + 6
t := x * y
```

$\longrightarrow$

Example: Copy Propagation + Constant Folding

```
a := 5
x := 2 * a
y := x + 6
t := x * y
```

$\longrightarrow$

```
a := 5
x := 10
y := 16
t := x << 4
```

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- Example (assume that a is not used anywhere else)

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x := z + y
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```
x := z + y            b := z + y
a := x       →        a := b       →
x := 2 * a            x := 2 * b
```

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```
x := z + y          b := z + y          b := z + y
a := x       →      a := b       →      x := 2 * b
x := 2 * a          x := 2 * b
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- Typically optimizations **interact**
  - Performing one optimization enables (or disables!) other optimizations
- Typical optimizing compilers repeatedly perform optimizations until no improvement is possible
  - **Phase ordering problem** again: must beware of local minima
- Interpreters and JITs must be fast!
  - The optimizer can also be stopped at any time to limit the compilation time

An Example

- Initial code:

```
a := x ** 2
b := 3
c := x
d := c * c
e := b * 2
f := a + d
g := e * f
```

An Example

- Algebraic simplification:

```
a := x ** 2
b := 3
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d := c * c
e := b * 2
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An Example

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An Example

- Copy propagation:

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a := x * x
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d := c * c
e := b + b
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```

An Example

- Copy propagation:

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a := x * x
b := 3
c := x
d := x * x
e := 3 + 3
f := a + d
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```

An Example

- Constant folding:

```
a := x * x
b := 3
c := x
d := x * x
e := 3 + 3
f := a + d
g := e * f
```

An Example

- Constant folding:

```
a := x * x
b := 3
c := x
d := x * x
e := 6
f := a + d
g := e * f
```

An Example

- Common subexpression elimination:

```
a := x * x
b := 3
c := x
d := x * x
e := 6
f := a + d
g := e * f
```

An Example

- Common subexpression elimination:

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b := 3
c := x
d := a
e := 6
f := a + d
g := e * f
```

An Example

- Copy propagation:

```
a := x * x
b := 3
c := x
d := a
e := 6
f := a + a
g := 6 * f
```

An Example

- Dead code elimination:

```
a := x * x
b := 3
c := x
d := a
e := 6
f := a + d
g := e * f
```

An Example

- Dead code elimination:

```
a := x * x
~~b := 3~~
~~c := x~~
~~d := a~~
~~e := 6~~
f := a + d
g := e * f
```

An Example

- Dead code elimination:

```
a := x * x
~~b := 3~~
~~c := x~~
~~d := a~~
~~e := 6~~
f := a + d
g := e * f
```

Could we get to `g = 12 * a`?

Local Optimization Notes

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- Intermediate code is helpful for many optimizations
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- "Program optimization" is grossly misnamed
  - Code produced by "optimizers" is not optimal in any reasonable sense
  - "Program improvement" is a more appropriate term

Course Announcements

- Graded **midterms** are at the front of the room
    - If you don't have it yet, pick it up now
    - If you take it with you, I won't accept regrade requests
- A problem with the PA3c3 autograder was found over the weekend
    - I've therefore granted an extension to **Wednesday** (AoE)
        - Same extension for PA3
- We recently fixed a bug in the reference compiler's x86-64 module. Only use Cool **version 1.39** for compiling to x86.

Example: Local Value Numbering

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