Functional Programming (1/2)

Martin Kellogg

Programming language paradigms

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  - **imperative**: change state, assignments
  - **structured**: if/block/routine control flow
  - **object-oriented**: message passing (=dyn. dispatch), inheritance
  - **functional**: functions are **first-class citizens** that can be passed around or called recursively. We can avoid changing state by passing copies.

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  - commands = instructions to the processor
  - array that is destructively updated = registers/memory/disk

Imperative programming: example

Consider the following C program:

```
double avg(int x, int y) {
  double z = (double)(x + y);
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  printf("Answer: %g\n", z);
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Consider the following C program:

semicolons separate commands, program is a list of commands

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destructive updates of memory cells

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    - "as powerful" = anything you can compute with a Turing machine can also be computed with the lambda calculus

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- functional programming **models math** well
  - it is easier to formally reason about functional programs

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    - Closures and **recursion**
    - **Lists** and list processing

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Let’s look at how imperative and functional languages **manage state** in a bit more detail

State management: functional vs imperative

Definition: The ***state*** of a program is all of the current variable and heap values

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- **Functional** programs yield **new similar states** over time.
  - `let x = y in …` , however, only changes `x`'s value **within** the scope of the …

Basic functional programming

Let's **translate** this C program into OCaml

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We'll come back to this later in the course, when we discuss **operational semantics**

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even the operators are type-safe (in OCaml)

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commands still exist, but limited to inherently "imperative" operations (I/O, saving to disk, etc.)

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no "return" statement, because everything is an expression

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tuple field extraction

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let y, z = x in
printf "1st element: %d" y;
...
```

point example:

```
let add_points p1 p2 =
  let x1, y1 = p1 in
  let x2, y2 = p2 in
  (x1 + x2, y1 + y2)
```

Lists are Your Friends

[Concept] [OCaml Syntax]

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Aside: “cons”, “car”, and “cdr”

- ***cons*** is a fundamental operation from Lisp, the first practical functional programming language (invented in the 1950s)
    - and 2nd higher-order language that’s still in use, after Fortran

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  - it’s used as shorthand for similar operations in modern FP
- you might also here “**car**” and “**cdr**” to refer to the first (resp. second) elements of a cons-pair (also historical Lisp terminology)

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All lists must be ***homogenous*** (i.e., all elements must **have same type**)

Functional examples

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- Simple function set (built out of lists):

```
let rec add_elem (s, e) =
    if s = [] then [e]
    else if List.hd s = e then s
    else List.hd s :: add_elem(List.tl s, e)
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```

- Same function using pattern matching instead:

```
let rec add_elem (s, e) = match s with
    | [] -> [e]
    | hd :: tl when e = hd -> s
    | hd :: tl -> hd :: add_elem(tl, e)
```

Equivalent Imperative (C) Code

```
List* add_elem(List *s, item e) {
  if (s == NULL) {
    return list(e, NULL);
  } else if (s->hd == e) {
    return s;
  } else if (s->tl == NULL) {
    s->tl = list(e, NULL);
    return s;
  } else {
    return add_elem(s->tl, e);
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}
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More cases to handle!

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- Tractable program semantics
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  - Formulate and prove assertions about code more easily
  - More readable (if you like math)
- ***Referential transparency***
  - Replace any expression by its value without changing the result
- “No” side-effects
  - Fewer errors

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Language	Speed	Space
C (gcc)	1.0	1.1
C++ (g++)	1.0	1.6
OCaml	1.5	2.9
Java (JDK -server)	1.7	9.1
Lisp	1.7	11
C# (mono)	2.4	5.6
Python	6.5	3.9
Ruby	16	5.0

17 small benchmarks

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- Compiler implementation
  - Frequent memory allocation
- Unfamiliar (to you!)
  - New programming style
- Not appropriate for every program
  - Some programs are inherently stateful (e.g., operating systems)

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Trivia Break: Computer Science History

This American computer scientist and mathematician was born in Washington, DC, in 1903. While a professor at Princeton, he advised Alan Turing's doctoral dissertation. He is known for inventing the lambda calculus, though he made many other contributions to mathematics, computer science, and philosophy.

Trivia Break: Cuisine

This dish is a sauce or gravy seasoned with spices, mainly derived from the interchange of Indian cuisine with European cuisine following the Columbian Exchange. Many types of this dish exist in different international cuisines. For example, in Southeast Asia, it often contains a spice paste and coconut milk. In India, the spices are fried in oil or ghee to create a paste. In Britain, this dish is regarded as national dish; some types were adopted from India, but others—such as Chicken Tikka Masala—were wholly invented in Britain in the 20th century.

ML’s innovative features

- Type system
  - Strongly typed
  - Type inference
  - Abstraction
- Modules
- Patterns
- Polymorphism
- Higher-order functions
- Concise formal semantics

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> *There are many ways of trying to understand programs. People often rely too much on one way, which is called "debugging" and consists of running a partly-understood program to see if it does what you expected. Another way, which ML advocates, is to install some means of understanding in the very programs themselves.*
>
> \- **Robin Milner**, 1997

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- If `x+3` has type `Int`, then `x+3` could evaluate to 7 or -2 or 5102 at run-time, but not "Hello" or 1.2
- To say that expression E has type T, we write:

  $$E : T$$

Types

Definition: A ***type*** is a conservative over-approximation of the set of values an expression could possibly take on at run-time.

- If `x+3` has type `Int`, then `x+3` could evaluate to 7 or -2 or 5102 at run-time, but not “Hello” or 1.2
- To say that expression E has type T, we write:

  E : T

- Types help us find bugs early
  - Requiring types to match up can rule out bad programs without even having to test them!

Aside: Why catch bugs earlier?

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- An IBM report gives an average defect repair cost of (2008$):
  - $25 during coding
  - $100 at build time
  - $450 during testing/QA
  - $16,000 post-release

[L. Williamson. IBM Rational software analyzer: Beyond source code. 2008.]

ML Type System

- Type Inference

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```
let rec add_elem (s, e) = match s with
    | [] -> [e]
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- Optional type declarations `( exp : type )`
  - Clarify ambiguous cases, documentation

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- ML infers (all!) types: inconsistent types are errors
- Optional type declarations (exp : type)
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You might be tempted to ask “How does ML infer types?” Unfortunately, this is a complex topic. Ask in OH if you’re curious, or take a PhD-level seminar from me or Iulian Neamtiu.

Pattern Matching

- Simplifies code (eliminates ifs, accessors)

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type btree = (* binary tree of strings *)
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let rec height tree = match tree with
   | Leaf _ -> 1
   | Node(x,_,y) -> 1 + max (height x) (height y)
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type btree = (* binary tree of strings *)
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   | Leaf of string
let rec height tree = match tree with
   | Leaf _ -> 1
   | Node(x,_,y) -> 1 + max (height x) (height y)
let rec mem tree elt = match tree with
   | Leaf str -> str = elt
   | Node(x,str,y) -> str = elt || mem x elt || mem y elt
```

Pattern Matching Mistakes

- What if I forget a case? E.g.,

```
let rec is_odd x = match x with
  | 0 -> false
  | 2 -> false
  | x when x > 2 -> is_odd (x-2)
```

Pattern Matching Mistakes

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   | 2 -> false
   | x when x > 2 -> is_odd (x-2)
```

```
Warning: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched: 1
```

Polymorphism

- Functions and type inference are ***polymorphic***

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let rec length x = match x with
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Recall that α means “any one type”

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length [1;2;3] = 3
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length [1;2;3] = 3
length [“algol”; ”smalltalk”; ”ml”] = 3
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length [1;2;3] = 3
length [“algol”; ”smalltalk”; ”ml”] = 3
length [1 ; “algol” ] = ?
```

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Recall that α means “any one type”

```
File "list-example.ml", line 1, characters 25-26:
1 | let myList = [ "algol" ; 1 ] in
                             ^
Error: This expression has type int but an expression was expected of type
         string
```

```
length [1 ; “algol” ] = ?
```

Higher-order functions

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f is itself a function!

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val map : (α -> β) -> ?
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val map : (α -> β) -> α list -> β list
let offset = 10 in
let myfun x = x + offset in
val myfun : ?
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Extremely powerful programming technique:

- general iterators
- implement abstraction

The Story of Fold

- We’ve seen **length** and **map**
- We can also imagine:

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or	[true; true; false]	= true
filter	(fun x -> x > 4) [1; 5; 8]	= [5; 8]
reverse	[1; 5; 8]	= [8; 5; 1]

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The Story of Fold

How can we **build** all of these?

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- We can also imagine:
    - **sum** [1; 5; 8] = 14
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- The *fold* operator comes from recursion theory (Kleene, 1952):

```
let rec fold f acc lst = match lst with
    | [] -> acc
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```

Note: acc type and return type are the same!

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```
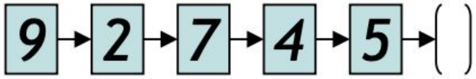

- on the whiteboard, this example (f is +):



Let’s build things out of fold

- **length** lst = fold (fun acc elt -> ???) ? lst

Let’s build things out of fold

- **length** lst = fold (fun acc elt -> acc + 1) 0 lst

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- **product** lst = <u>fold</u> (fun acc elt -> acc * elt) 1 lst

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- **and** lst = <u>fold</u> (fun acc elt -> acc & elt) true lst

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- think you can do **or** on your own?

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- think you can do **or** on your own?
  - what about **reverse**?

Let’s build things out of fold, part 2

- **reverse** lst = <u>fold</u> (fun acc elt -> ???) ? lst

Let's build things out of fold, part 2

- **reverse** lst = fold (fun acc elt -> acc @ [e]) [] lst

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- **reverse** lst = <u>fold</u> (fun acc elt -> acc @ [e]) [] lst
  - note types: **(acc : α list) (e : α)**

Let’s build things out of fold, part 2

- **reverse** lst = <u>fold</u> (fun acc elt -> acc @ [e]) [] lst
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- **filter** keep_it lst = <u>fold</u> (fun acc elt -> ???) ? lst

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  - note types: (acc : bool) (e : α)

Let's build things out of fold, part 2

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- Could we do **map**?
  - Recall: map (fun x -> x +10) [1;2] = [11;12]

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- Could we do **map**?
  - Recall: map (fun x -> x +10) [1;2] = [11;12]
  - Let's do it together…

Let's build things out of fold, part 3 (map)

```
let map myfun lst =
        fold (fun acc elt ->          ???        ) ? lst
```

Let's build things out of fold, part 3 (map)

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let map myfun lst =
        fold (fun acc elt -> (myfun elt) :: acc) [] lst
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Let’s build things out of fold, part 3 (map)

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- Types of:
  - myfun : **α -> β**
  - lst : **α list**
  - acc : **β list**
  - elt : **α**

Let's build things out of fold, part 3 (map)

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- Types of:
  - myfun : **α -> β**
  - lst : **α list**
  - acc : **β list**
  - elt : **α**
- Could we do **sort**?

Sorting examples

let **langs = [“fortran”; “algol”; “c”]** in

- <u>sort</u> (fun a b -> ???) langs = **[“algol”; “c”; “fortran”]**

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- sort (fun a b -> a > b) langs = [“fortran”; “c”; “algol”]

Sorting examples

let **langs = [“fortran”; “algol”; “c”]** in

- <u>sort</u> (fun a b -> a < b) langs = [“algol”; “c”; “fortran”]
- <u>sort</u> (fun a b -> a > b) langs = [“fortran”; “c”; “algol”]
- <u>sort</u> (fun a b -> ???) langs = **[“c”; “algol”; “fortran”]**

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- sort (fun a b -> a < b) langs = [“algol”; “c”; “fortran”]
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- sort (fun a b -> strlen a < strlen b) langs = [“c”; “algol”; “fortran”]

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- Recall Java’s **Comparator** interface
  - in this functional style, our implementations are much simpler!

Partial Application and Currying

```
let myadd x y = x + y
val myadd : int -> int -> int
myadd 3 5 = 8
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- How do we know what this means? We use ***referential transparency***! Basically, just substitute it in.

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- How do we know what this means? We use ***referential transparency***! Basically, just substitute it in.

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val addtwo : int -> int
addtwo 77 = 79
```

- called ***Currying***: "if you fix some arguments, you get a function of the remaining arguments"

Course Announcements

- Don’t forget: PA1c1 **due today**
  - and PA1c2 (1 more language!) **due Thursday**
  - and PA1 (full, all four languages!) **due next Monday**
- Cool Reference Manual is assigned reading for Wednesday, too ;)
  - I ***certainly*** wouldn’t consider giving another quiz…

Course Announcements