Global Optimization (2): Dead Code Elimination

Martin Kellogg

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  - recall that PA4 is due on the day of the final exam
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- There was a bug in the reference compiler's implementation of the `in_string()` builtin
  - new version of Cool (v1.40) released
  - I will not test the difference between the two, so it's okay to continue to use v1.39

Agenda

- Global constant folding
- Global liveness analysis
  - this analysis enables dead code elimination
- Interprocedural optimizations (and analysis)

Review: Global Constant Folding

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- To replace a use of *x* by a constant *k* we must know this *correctness condition*:

  On every path to the use of x, the last assignment to x is x := k
- This correctness condition is **not trivial** to check
  - "**All paths**" includes paths around loops and through branches of conditionals

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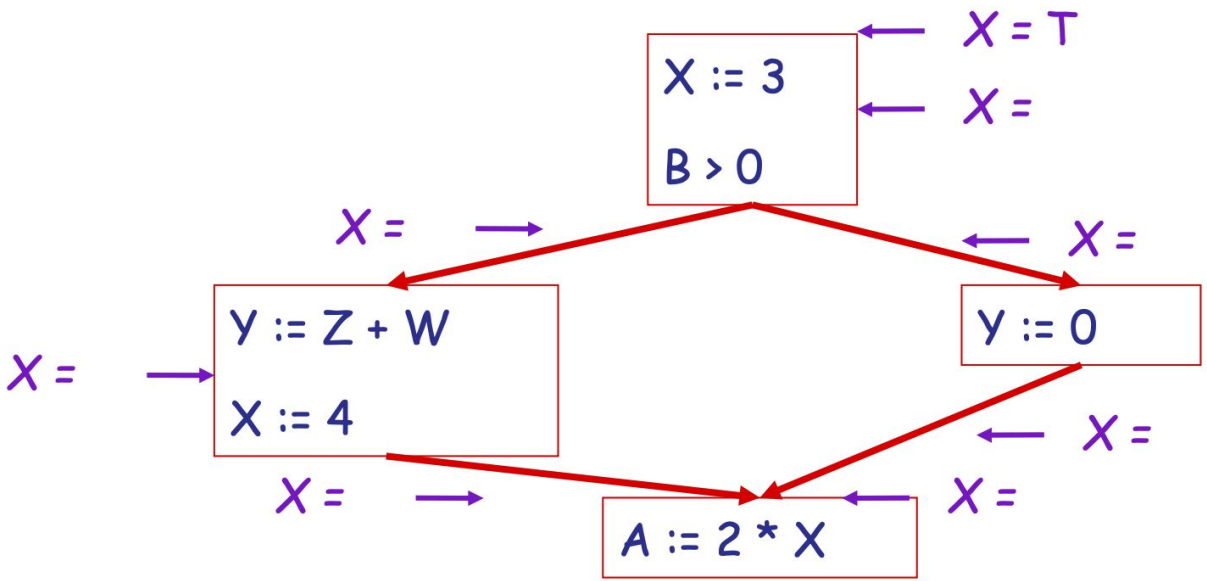
  - we can’t check the correctness condition directly because it is **undecidable** (as a direct corollary of Rice’s Theorem)
- Given global constant information, it is **easy to decide** whether or not to perform the optimization

Review: Global Constant Folding Abstraction

- To make the problem precise, we associate one of the following **abstract values** with X at every program point:
  - T (“top”) = “don’t know if X is a constant”
  - constant *c* = “the last assignment to X was X = c”
  - ⊥ (“bottom”) = “X has no value here”

Global Constant Folding: Formalized

Get out a piece of paper. Fill in these blanks:



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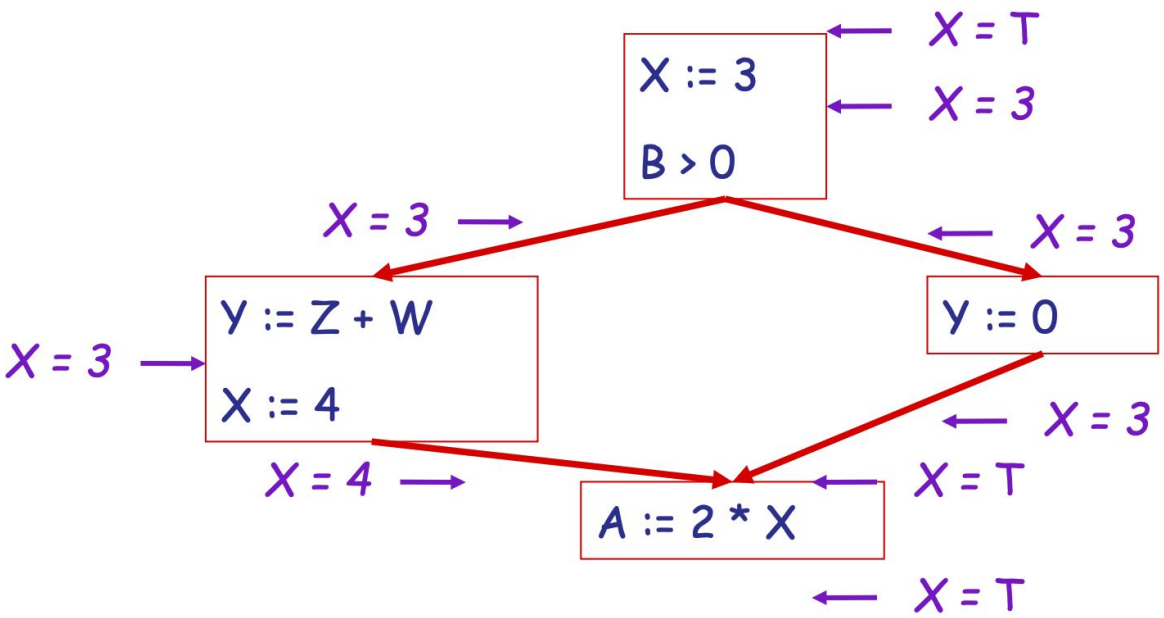

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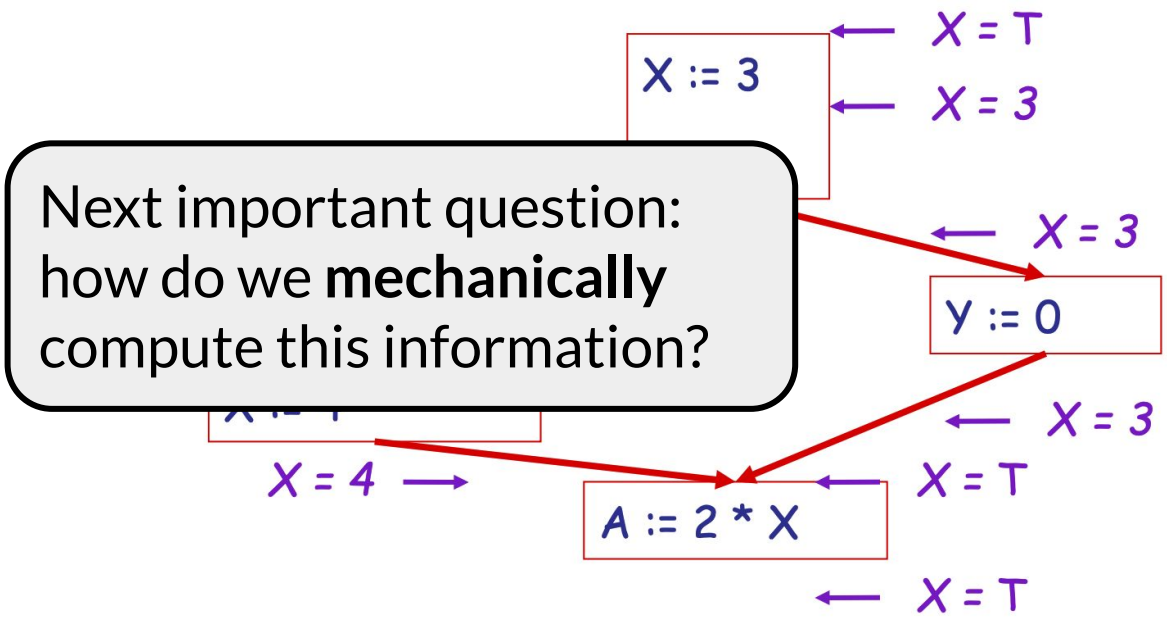

Next important question: how do we **mechanically** compute this information?

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Key Idea of Dataflow Analysis

The analysis of a complicated program can be expressed as a combination of simple rules relating the change in information between adjacent statements

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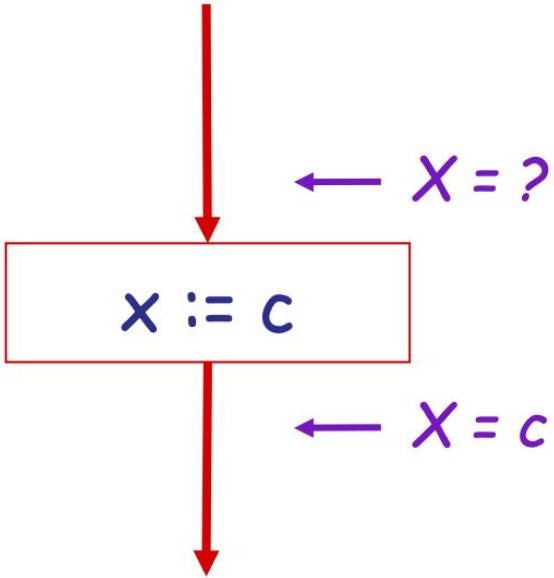
Definition: a ***transfer function*** expresses the relationship between $C_{in}(x, s)$ and $C_{out}(x, s)$

Transfer functions: rule 1

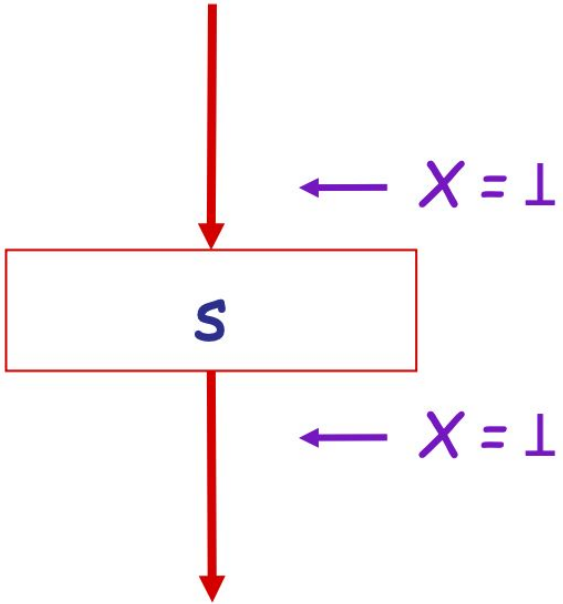


$C_{out}(x, x := c) = c$ if c is a constant

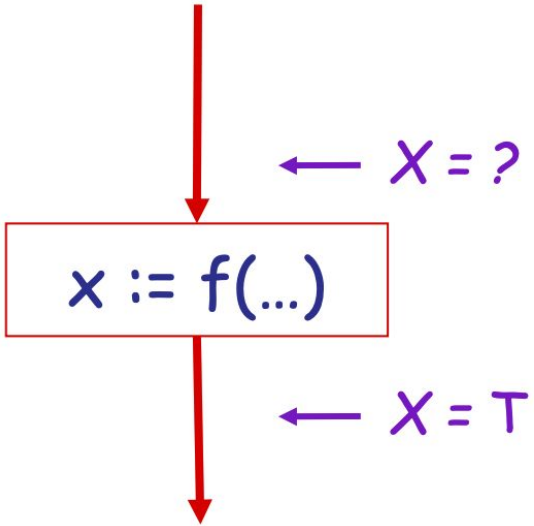
Transfer functions: rule 2

$C_{out}(x, s) = \text{bottom if } C_{in}(x, s) = \text{bottom}$

Recall bottom = “unreachable code”



Transfer functions: rule 3



$C_{out}(x, x := f(...)) = T$

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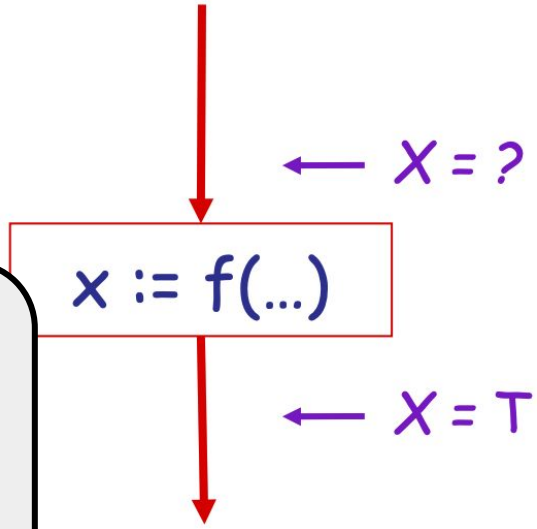
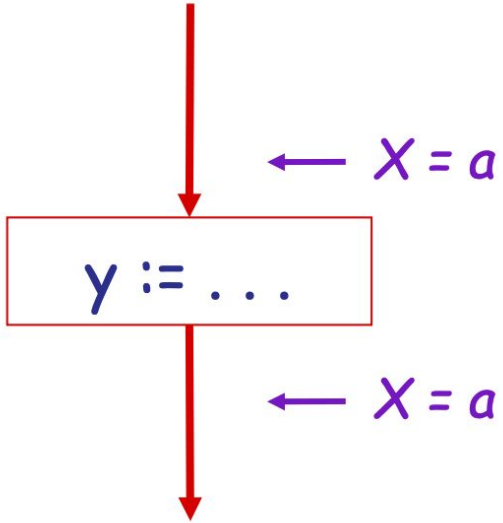


This is a **conservative approximation**! f(...) might always return a constant, but we don't even try!

$C_{out}(x, x := f(...)) = T$

Transfer functions: rule 4



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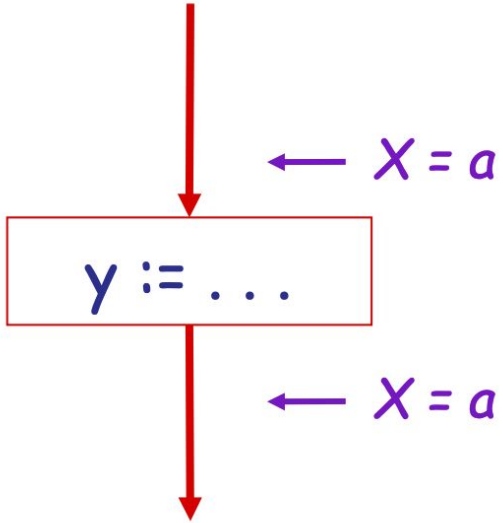

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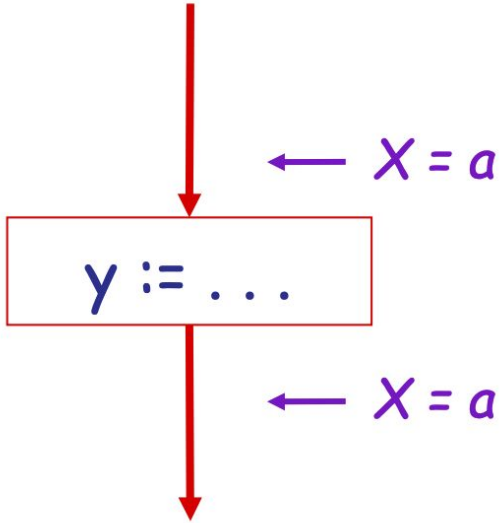
How hard is it to check if x ≠ y on all executions? (oh no)



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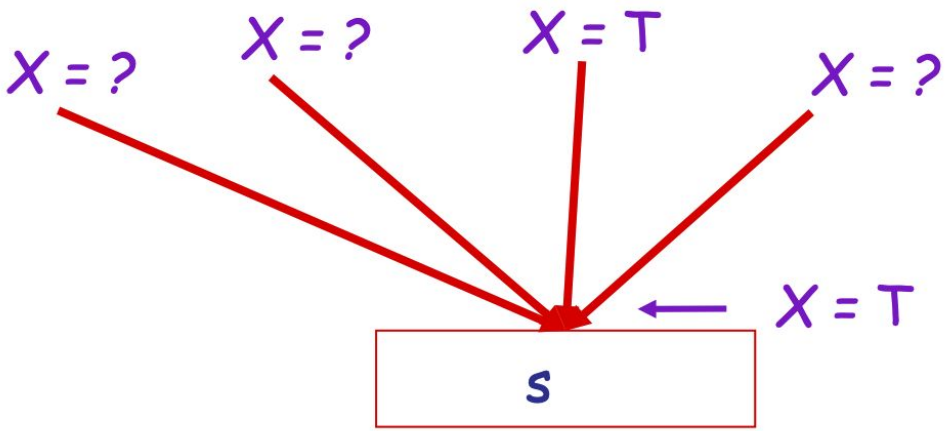
- In the following rules, let statement s have immediate predecessor statements p_1 , …, p_n

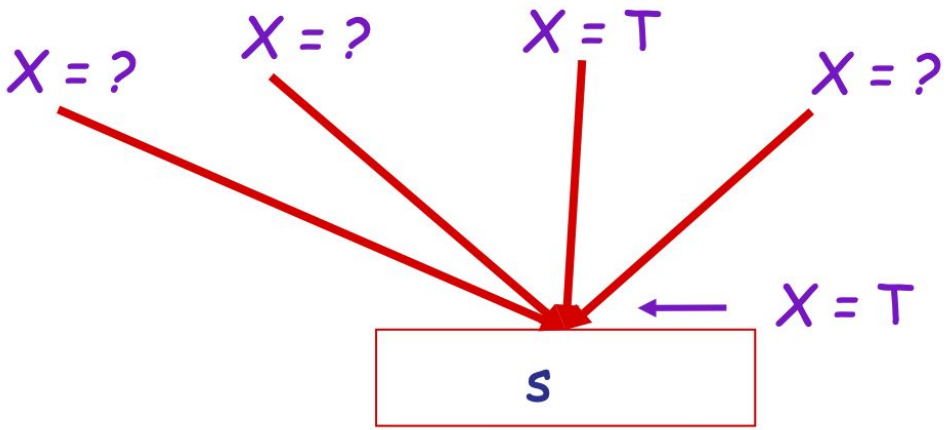
Transfer functions: rule 5



if $C_{out}(x, p_i) = T$ for some i, then $C_{in}(x, s) = T$

Transfer functions: rule 5

If there's **any** path on which we don't know, then we don't know at all



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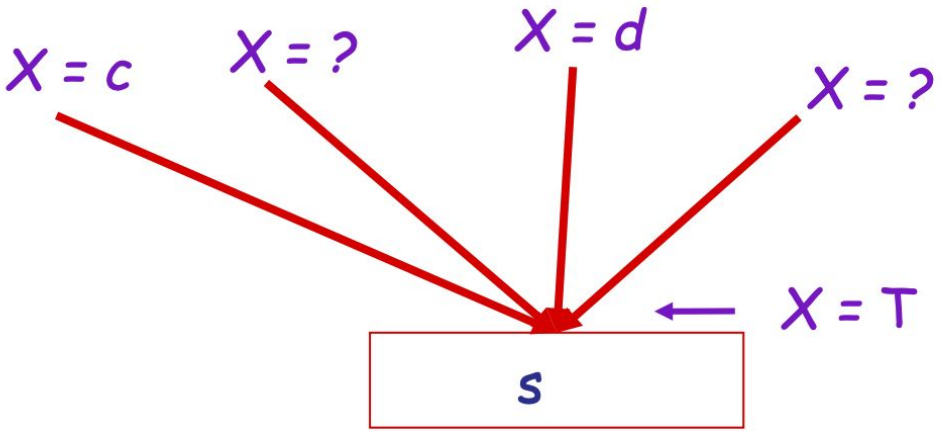

if $C_{out}(x, p_i) = c$ and $C_{out}(x, p_j) = d$ and $d \neq c$ then $C_{in}(x, s) = T$

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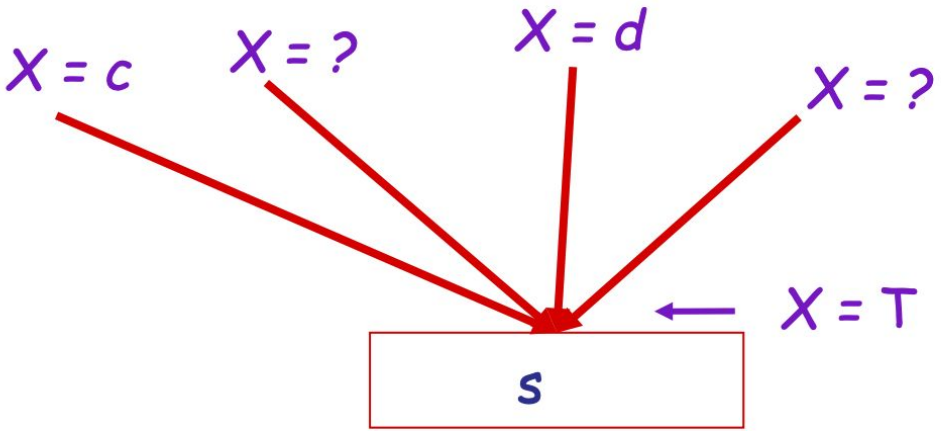

We **don't know** which of the paths a given execution will take (so assume T)

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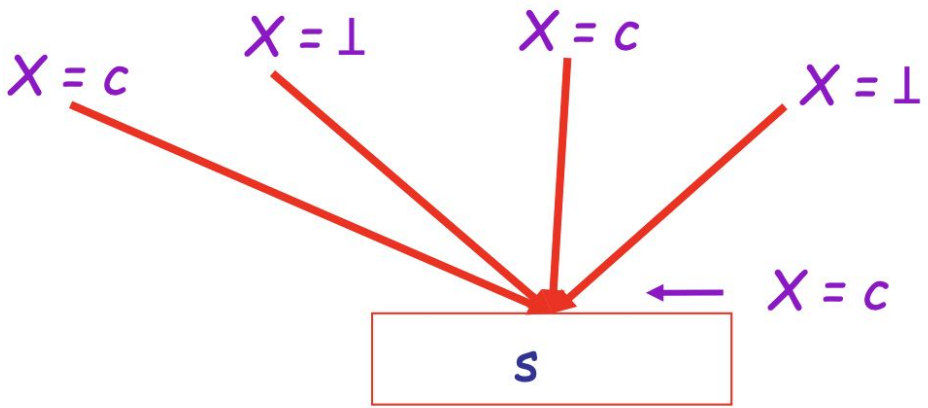
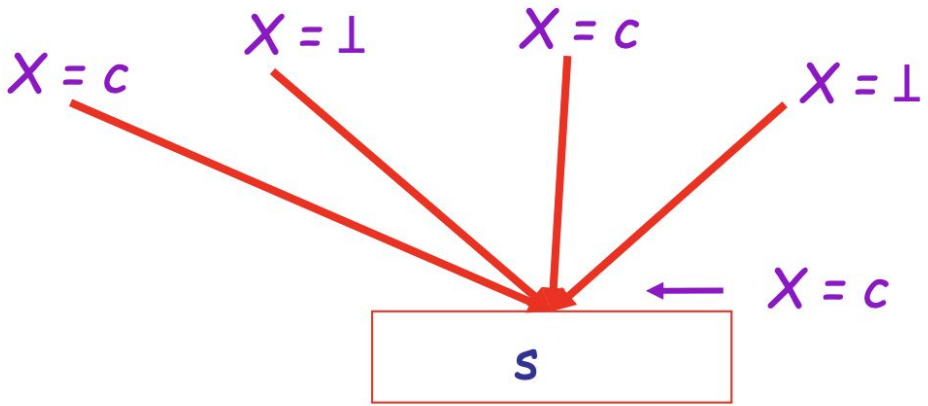
Transfer functions: rule 7



if $C_{out}(x, p_i) = c$ or bottom for all i, then $C_{in}(x, s) = c$

Transfer functions: rule 7

If x has the **same** value (or bottom) on all input edges, it has that value in s



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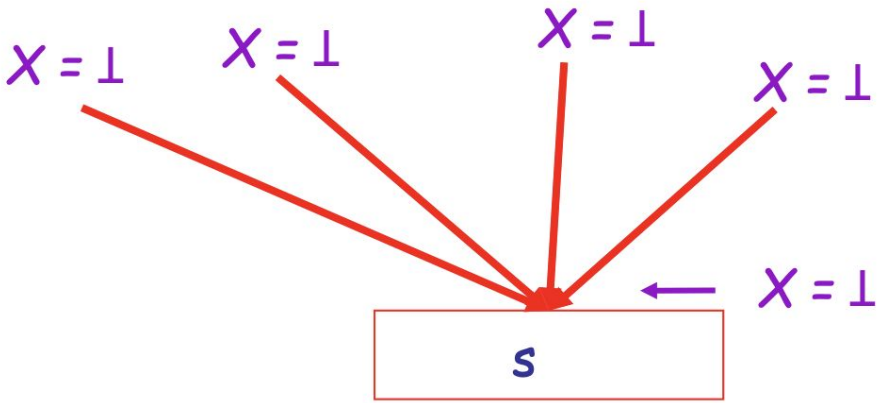

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This is a ***fixpoint*** (or ***fixed point***) ***iteration*** algorithm. Such algorithms are characterized by a finite set of rules, which are applied until they "reach fixpoint", which means that applying any rule produces no change.

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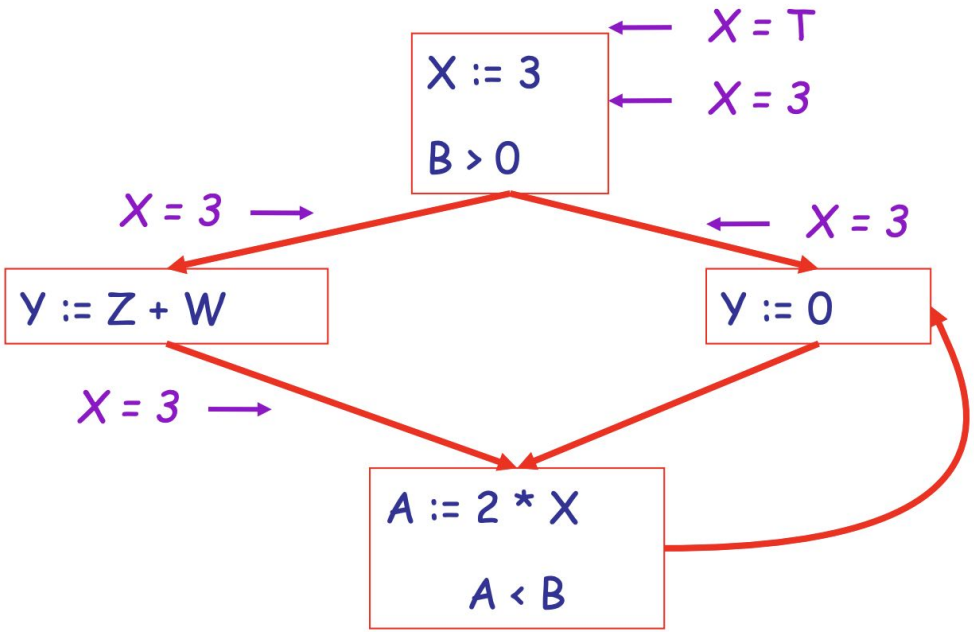

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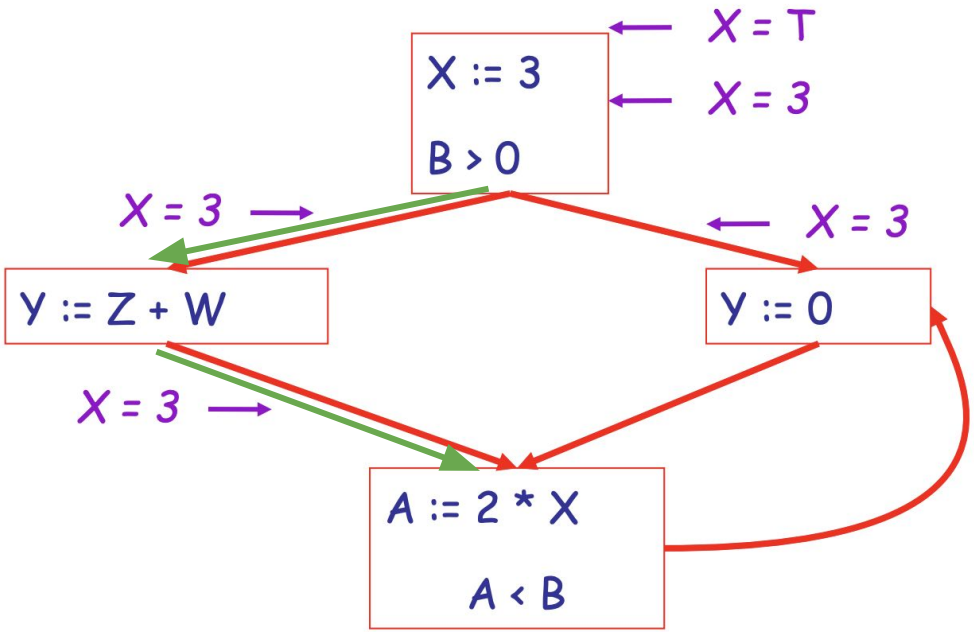

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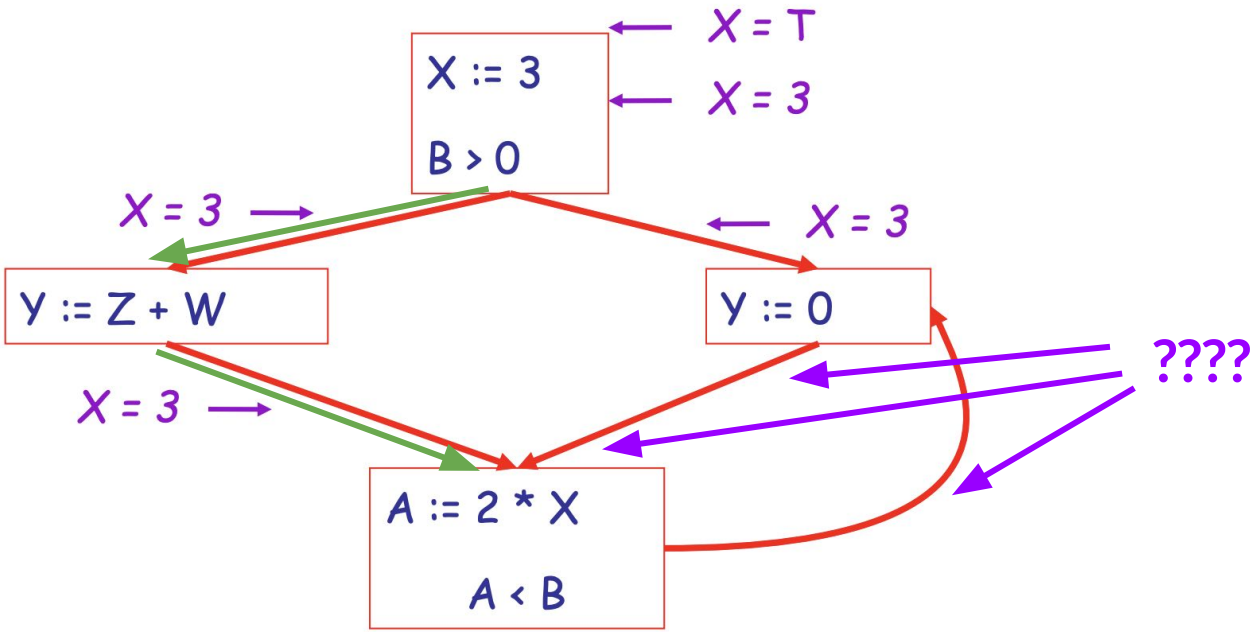

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- Because of **cycles**, all points must have values at all times during the analysis
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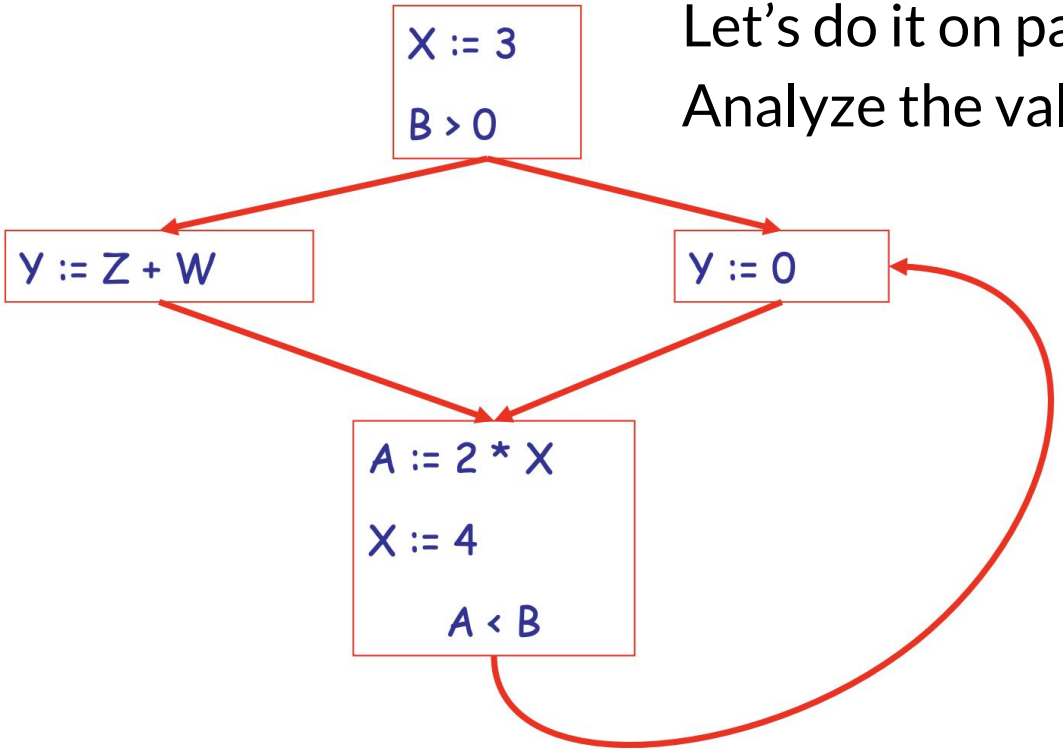
- The initial value bottom means "**we have not yet analyzed control reaching this point**"

Another example: dealing with loops



Let's do it on paper!
Analyze the value of X.

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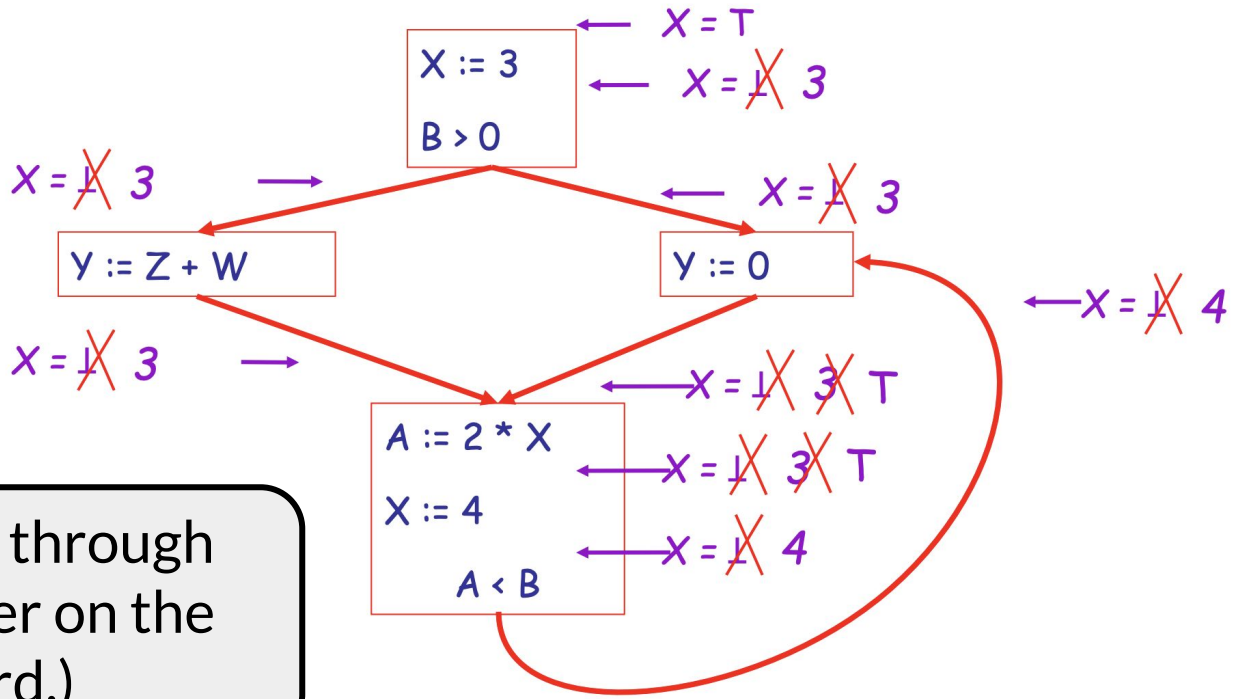

(We went through this answer on the whiteboard.)

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  - Locations whose current value is T never change

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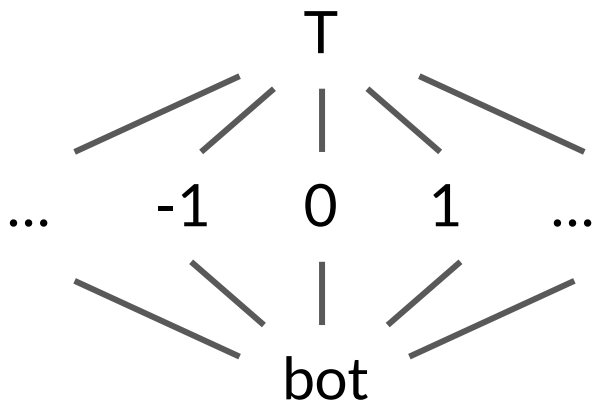
This structure between values is a ***lattice*** (just like in AI!):



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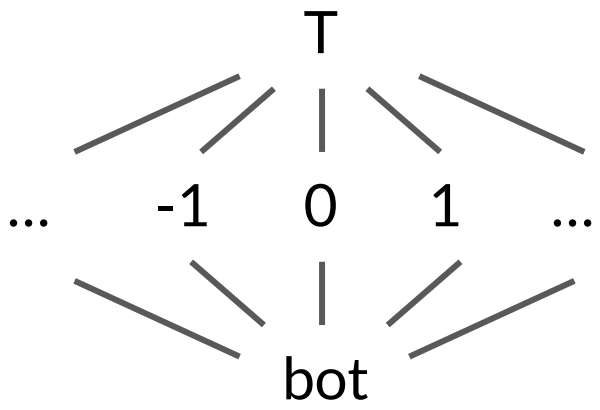

Review of how to read a lattice:

- abstract values **higher** in the lattice are **more general** (e.g., T is true of more things than 0)
- easy to compute ***least upper bound***: it's the lowest common ancestor of two abstract values

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lub is the reason dataflow analysis is an **algorithm**: because lub is monotonic, we only need to analyze each loop **as many times as the lattice is tall**

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- The use of **lub** explains why the algorithm terminates:
  - values start as bottom and only increase
  - bottom can change to a constant, and a constant to T
  - thus, C_(x, s) can change at most **twice** (= lattice height minus one)

Trivia Break: Music

BBC disc jockey John Peel said of this 1969 experimental rock album, "If there has been anything in the history of popular music which could be described as a work of art in a way that people who are involved in other areas of art would understand, then [this album] is probably that work." Its unconventional musical style, which includes polyrhythm, and polytonality, has given the album a reputation as one of the most challenging recordings in the 20th century musical canon. The album was recorded in a single six-hour session after the band had rehearsed it for eight months (!!) straight.

Name either the album or the band that created it.

Trivia Break: Art History

This early-20th-century avant-garde art movement that began in Paris revolutionized painting and the visual arts, and influenced artistic innovations in music, ballet, literature, and architecture. Its subjects are analyzed, broken up, and reassembled in an abstract form: instead of depicting objects from a single perspective, the artist depicts the subject from multiple perspectives to represent the subject in a greater context. Important artists in the movement include Pablo Picasso, Georges Braque, Jean Metzinger, Albert Gleizes, Robert Delaunay, Henri Le Fauconnier, Juan Gris, Fernand Léger, and others.

Picasso's *Girl with a Mandolin (Fanny Tellier)*

Liveness Analysis

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  - An “abstract interpretation”-like one (transfer functions)
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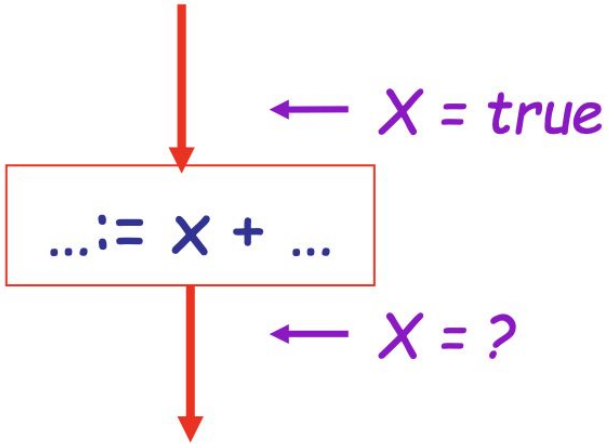
- These formalisms are **equally valid**. You can use which makes the most sense to you when you’re implementing PA4c1/on exams/etc

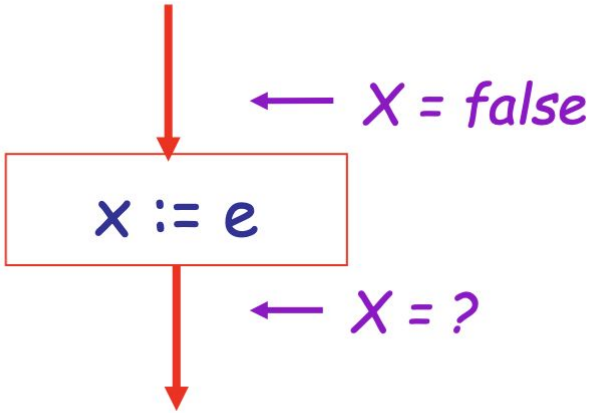
Liveness Analysis: As Rules

Liveness Analysis: Rule 1



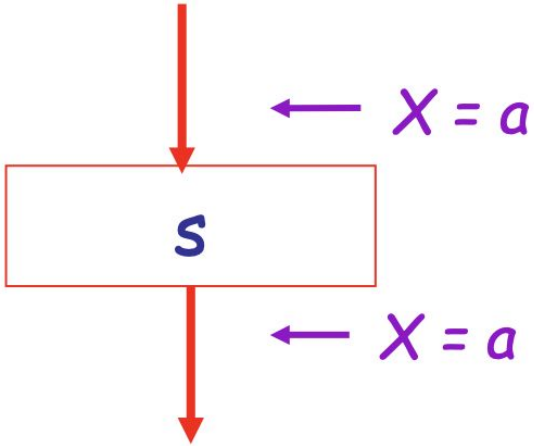
$L_{in}(x,s)$ = true if s refers to x on the rhs

Liveness Analysis: Rule 2



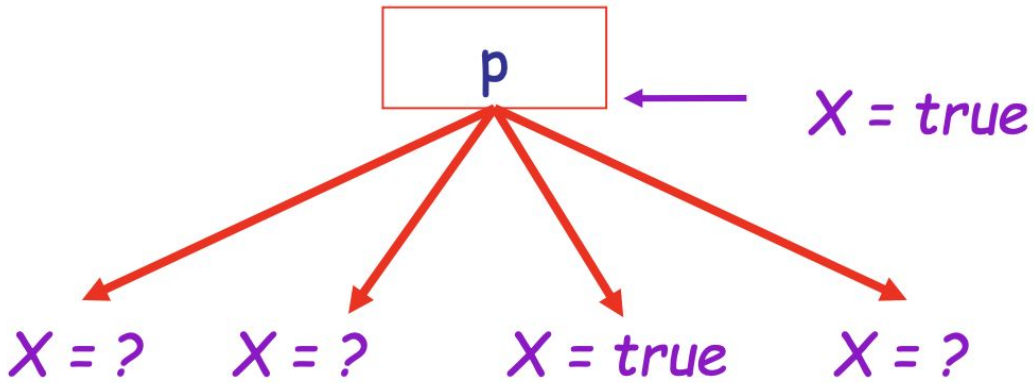
$L_{in}(x, x := e) = false$ if e does not refer to x

Liveness Analysis: Rule 3



$L_{in}(x, s) = L_{out}(x, s)$ if s does not refer to x

Liveness Analysis: Rule 4



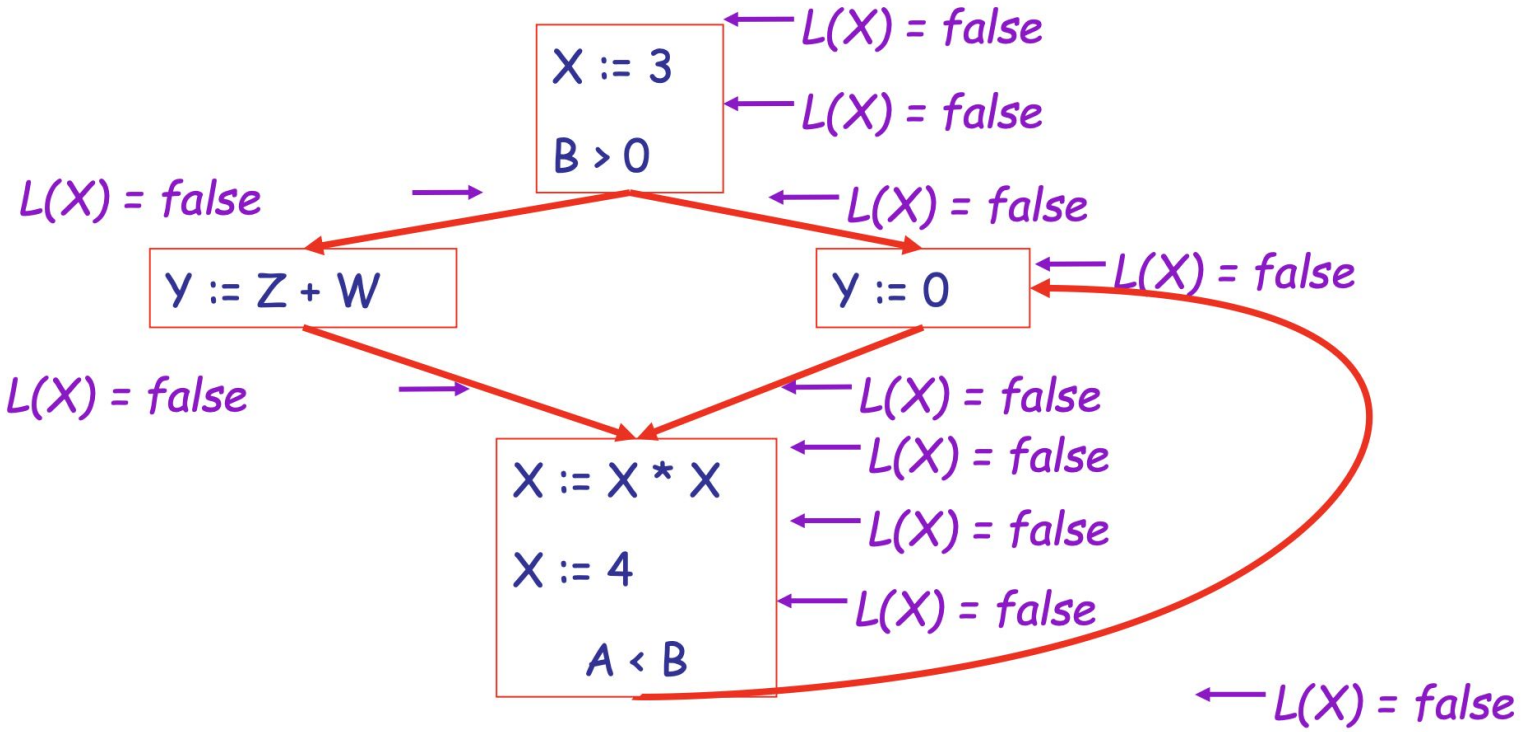
$L_{out}(x, p) = \bigvee \{ L_{in}(x, s) \mid s \text{ is a successor of } p \}$

Liveness Analysis: Algorithm

- Let all L_(...) = false initially
- Repeat until all statements s satisfy rules 1-4 :
  - Pick s where one of rules 1-4 does not hold and update using the appropriate rule

(on the whiteboard)

Liveness Analysis: Example



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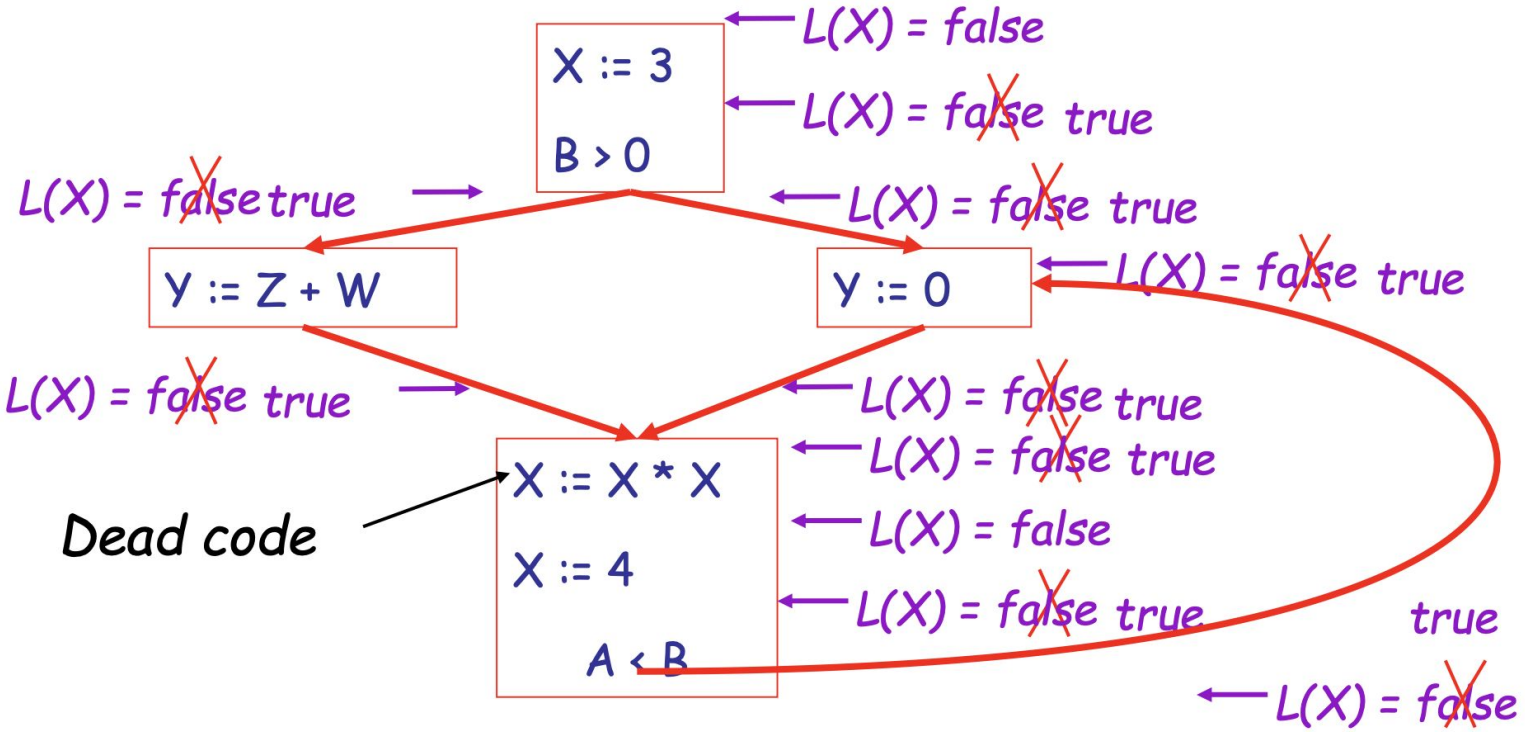

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Alternative algorithm for dataflow analysis: solve this ***dataflow equation*** iteratively for all blocks until all sets reach fixpoint

Liveness Analysis: Dataflow Equations

- To use this framework to model liveness, we need to define these four sets:
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  - a variable **becomes live** because of a use
  - a variable is **no longer live** because of a definition

Dataflow Equations vs Transfer Functions

- Either way of thinking about dataflow analysis is **equally valid**

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Dataflow Equations vs Transfer Functions

- Either way of thinking about dataflow analysis is **equally valid**
  - I prefer transfer functions, but you might prefer dataflow equations
  - We can easily prove that anything that we can express with one is can be expressed with the other
- There is a much more in-depth treatment of dataflow equations (including how to use them for liveness analysis) in *EaC* chapter 9.
  - If you're confused, **start there**

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  - **Register allocation**: only live variables need a register
  - Detecting uses of **uninitialized variables**: if live at declaration (before initialization) then it might be used uninitialized
  - Improve **SSA construction**: only need Φ-functions for variables that are live in a block (later)

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Liveness Analysis: Summary

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- **Two ways** to do liveness analysis (equally-valid!):
  - transfer functions
  - dataflow equations
- Liveness analysis has lots of uses:
  - DCE, register allocation, detecting uses of uninitialized variables, etc.

Interprocedural Optimizations

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Today we will take a brief look at two interprocedural optimizations:

- inlining
- tail call optimization

Interprocedural Optimizations: Inlining

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- Key idea of ***inlining***: for such a procedure call, replace the call with the procedure's body

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Note similar benefits and risks to **loop unrolling**. Deciding whether to inline is similarly complicated!

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  - **Profile data**, such as fraction of execution time (if available)

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    - Including many **redundant operations** (e.g., resetting `%rsp`)

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- **Functional languages** practically require tail-call elimination

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  - Traditional "***compilation unit***" is a procedure or file
- If all of the code is definitely available, it suffices to **track dependencies** between procedures from an optimization perspective, and then re-optimize whenever a dependent procedure changes
- Alternatively, we can defer interprocedural optimization until ***link time***, when a ***linker*** combines the object files from each compilation unit into a single executable. We'll talk more about this later.

Course Announcements

- PA3 deadline is now Monday, April 14 AoE
  - I have also moved the PA4c1 deadline to 4/28
- PA4 leaderboard will go live (with your PA3 submissions so far) Sometime Soon™
  - recall that PA4 is due on the day of the final exam
    - which means I cannot grant extensions on it!
- There was a bug in the reference compiler's implementation of the `in_string()` builtin
  - new version of Cool (v1.40) released
  - I will not test the difference between the two, so it's okay to continue to use v1.39